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GODEFRIDI ANT. DÉCORÉ,

ARITHMETICA UNIVERSALIS SUMMI NEWTONI,

Contracta, Illuſtrata & Locupletata.

Præeunte.

LOGICA ANALITICA.

UNIV. OF MICH.

LUGDUNI BATAVORUM,
Apud CORNELIUM DE PECKER,
MDCCLXI.

CELSISSIMO ET *SERENISSIMO*

WILHELMO V.

ARAUSIONIS ET NASSAVIÆ
PRINCIPI, ETC. ETC. ETC.

TOTIUS FOEDERATI BELGII
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Cel-

CELSISSIMO ET POTENTISSIMO PRINCIPI

Omnibus quidem bonis, ſed imprimis tibi Heroum filio, ſpei patriæ, offerendus erat Heros Litteratorum, qui propter miram mentis ſublimitatem junctam pari morum Candori meruit Weſtmonaſterii cum Regibus Patribus tuis ſepeliri. Filios Magnatum debet eſſe viros dum adhucdum adoleſcunt cæteri, nec Inclitæ Genti noſtræ deſunt indicia quibus Te PRINCEPS AUGUSTE talem afflante Numine futurum auſpicetur, dum tuis in reſponſis ſapientiæ ſalem, in jocis moderationem, in ſcientiis addiſcendis ſedulitatem, in Memoria tenacitatem, in moribus Animam bo-

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bonam mirantur & deprædicant, domi foriſque, Teſtes occulatiſſimi.

Quis dubitet quin SERENISSIMO DUCE *Tutore, Principum ſapientiſſimo, cujus ſollicitudo primaria eſt exemplo quotidiano illud audiri poëtæ.*

Diſce Puer virtutem ex me, verumque Laborem.

Quis inquam dubitet, quin rapido curſu virtutum omnium tam intellectualium quam moralium culmen confeſtim attingas & ita ſupplicationes Eccleſiæ ab Auctore omnium bonorum exaudiantur?

Quo Leviori pede ſtadium illud nobile pariter & arduum percurri queat, decet Exercitatos ſimbolam,

lam, quemque ſuam, conferre & ſpinas obvias averruncare. Ea ſpe fretus, ſub magni Tui nominis umbra PRINCEPS SERENISSIME, *te annuente, prodit qua carere vix poteſt Matheſeos Cultor*, ARITHMETICA UNIVERSALIS SUMMI NEWTONI, *contracta quidem ſed illuſtrata & locupletata, ut intelligibilior evadat liber aureus. Verum intelligere nemo poteſt, niſi qui pectoris intima diſcernit, vota quæ, devotiſſima mente, pro ſalute & ſoſpitate* SERENISSIMÆ CELSITUDINIS TUÆ, *quæ vera populi liberi felicitas eſt, effundit*

CELSISSIME ET SERENISSIME PRINCEPS!

Cliens addictiſſimus

GODEFRIDUS ANT. DECORÉ.

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GODEFRIDI ANT. DÉCORÉ, ARITHMETICA UNIVERSALIS SUMMI NEWTONI,

Contracta, Illuſtrata & Locupletata.

Præeunte.

LOGICA ANALITICA.

I. Unum erat inter deſiderata *Verulamii*, Philoſophos modernos eo ſcopo collimare, quo tandem finis imponeretur veteri quærelæ de *Arte longa & Vita brevi.* Annuens placito viri ſagaciſ-

ſimi contrahere ſimul & illuſtrare conſtitui *Arithmeticam Univerſalem* Newtoni, ſequendo ductum Logicæ, ſat naturalis quidem ſed ſeverioris, quæ ideas medias Demonſtrationum, ad Euclidis modum, ex aliquot *poſtulatis*, *axiomatibus* & *definitionibus* ubique depromat. Hoc pacto fiet ut liber exoptatiſſimus, nunc rariſſimus, mole parvus, materia plenus, in exemplis certo conſilio electis longus (ut minus dicam) triplo brevior adhuc & in ratione directa brevitatis clarior evadat. Indicem capitum XXV Editioni s'Graveſandianæ præfixum ſequar. Quæſtiones omnes Newtoni tam Arithmeticas XVI, quam Geometricas LXI, hasque cum Tabulis & figuris tradam. Demonſtrationes quas (quod monet in appendice) ut pote *ſatis faciles ſibi viſas* omiſit, ſine ambage ſupplebo. Caput penultimum, quouſque declarat Auctor, (*pag.* 208.) *perexigui uſus eſſe*, ut & appendicem, majori cum fructu & utilitate illuſtrabo per ea quæ habet in *Opuſculis* & quæ pro reſolutione omnium æquationum per ſeries infinitas, in *Arithmetica Univerſali* deſiderantur. Doctrinam Combinationum, Progreſſionum, nonnullaque omiſſa ſuperaddam. Si quis autem in his me, hominem veritatis amicum, ſua corre-

ctione

ctione dignabitur., *utinam ipſe lucernam meam caligantem ſole ſuo lucidiore extinguat?*

LOGICA ANALITICA.

2. Hæc naturalis valde. Logicam naturalem appello vim illam mentis qua homo directe & ſine ſaltu demonſtrare valet, *ex. gr.* annulum ultimum Catenæ pendulæ ab annulo primo ſuſtentari. Quod quam facile ſit nemo non videt, ſive per *ſoritem* uno tractu dicam ,, annulus *primus* ſuſtentat ſecun,, dum, ſecundus tertium, tertius quar,, tum &c. Ergo *primus ultimum*" ſive per tot proſillogiſmos quot dantur annuli inter extremos medii, dixero ,, *annulus* ,, *primus ſuſtentat* ſecundum; ſed ſecundus ,, *ſuſtentat ultimum:* Ergo *annulus primus* ,, *ſuſtentat ultimum.*" Majorem ſeu primam propoſitionem pudor negare vetat: quia propoſitio eſt *immediatæ veritatis*, hoc eſt, quia non datur annulus medius inter 1um & 2um.: hinc, ut probetur minor, dico ,, *annulus ſecundus* ſuſtentat ter,, tium, ſed tertius ſuſtentat *ultimum: Er,, go ſecundus* ſuſtentat *ultimum.*" Ita gradatim pergo, donec perveniam ad annulos tres ultimos ex quibus fiet ſillogismus, cujus major & minor erunt propoſitio-

nes *immediatæ veritatis* ; ſcilicet ,, *annulus*
,, *antipenultimus* ſuſtentat penultimum; ſed
,, penultimus ſuſtentat *ultimum:* ergo *anti-*
,, *penultimus* ſuſtentat *ultimum.*" Quæ cum ſequantur ex prius enuntiatis , ſequitur ea omnia eſſe vera ; quia verum ex falſo nequit gigni , ut nec ex vero falſum.

3. Ex hac Analogia catenæ methodus intelligitur demonſtrandi propoſitiones univerſales affirmativas quaſcunque (quales ſunt omnes in *Analiſi ſpeciosa*) quin & negativas. Quippe eo modo quo fit unus ſillogismus & una demonſtratio , ita omnes in quacumque ſcientia. Hinc quemadmodum erant in priori ſillogismo tres annuli , *primus* , ſecundus & *ultimus* , ita ſunt in omni ſillogismo tres ideæ ſubordinatæ , *extremæ* duæ & una *media* , quarum collatio facit tres propoſitiones: Omnis propoſitio eſt neceſſario vera vel falſa , contingenter vero affirmativa vel negativa : hinc omnis negativa in affirmativam converti poteſt. Propoſitio eſt affirmatio ,, quod idea una contineat aliam
,, vel ejus excluſionem." Idea continens eſt *ſubjectum* & idea contenta *prædicatum* propoſitionis. Prima propoſitio ſillogismi vulgo vocatur *major* , ſecunda *minor* & tertia *conſequentia* , ſeu reſponſum ad quæſtionem. Major & minor ſimul *præ-*

miſſæ

miſſæ dicuntur. Hinc primo neceſſe eſt ſtatum quæſtionis in mente firmiter teneri, ne plus in reſponſo quam ſit in quæſito ponatur; dein ideam mediam quæri. Hac inventa ſillogismus ſe ſponte prodit. Eſt autem *legitimus*, *dum idea media in una extremarum continetur & alteram continet*. Sic in exemplo catenæ, annulus *medius* eſt qui ſuſtentatur a primo & ultimum ſuſtentat. In hac ſola regula eſt omnis *Energia* ſillogismi. Qui hanc ſequitur nunquam errat, qui ab ea diſcedit vel *Pſeudo-ſillogismum* vel *Sophisma facit*.

4. PSEUDOSILLOGISMUS eſt in cujus præmiſſarum una vel ambabus, idea ſubjecti non continet id quod de ipſa prædicatur. Conceſſis illis, conſequentia quamvis formaliter falſa negari nequit.

SOPHISMA eſt in cujus præmiſſis idea *media* collata cum extremis vel bis continet vel bis continetur. Conceſſis illis conſequentia negari poteſt.

Hinc non poteſt fieri ſillogismus ex tribus ideis quibuscunque, niſi fuerint ſubordinatæ. Imo ex datis tribus ideis ſubordinatis poſſunt ſemper confici duo ſophismata & unus dumtaxat ſillogismus legitimus Dico „ *annulus* 1us ſuſtentat 2um „ ſed *annulus* 3us ſuſtentat 2dum. Ergo *annulus* 1us ſuſtentat 3um." Hoc eſt ſophis-

ma; nam idea media bis continetur. Iterum dico, „ annulus 2^{dus} ſuſtentat 1um: ſed „ annulus 2^{dus} ſuſtentat 3um. Ergo annulus „ 1us ſuſtentat 3um." Hoc eſt ſophisma; nam idea media bis continet. Porro dico „ *annulus* 1us ſuſtentat 2^{dum}; ſed 2^{dus} „ ſuſtentat 3um: Ergo 1us ſuſtentat 3um" & recte demum concludo; quia idea media ſemel continet & ſemel continetur.

Enthymema eſt ſillogismus cujus una præmiſſarum ſubticetur. Quod perquam utile eſt, ubi ſupprimitur ea quæ minus immediatæ veritatis eſt: ut (2) *annulus primus* ſuſtentat 2^{dum}: Ergo *ultimum*.

5. Omnis propoſitio demonſtrabilis eſt, ſi non per *principia directa*, ſaltem per *principium* reflexum *contradictionis*, hoc ipſo ſolo excepto, quod ſic effertur: *Impoſſibile eſt idem ſimul eſſe & non eſſe*. *v. g. Qtr*, an annulus primus catenæ ſuſtentet 2um? Reſp. impoſſibile eſt idem ſimul eſſe & non eſſe; ſed annulus 1us foret & non foret 1us ſi non ſuſtentaret 2^{dum}: Ergo annulus 1us ſuſtentat 2um. Major tam evidens eſt, ut quo plus velis demonſtrare eo obſcurior fiat; minor per partes probatur *ad hominem* agnitæ veritatis impugnatorem, eo dumtaxat fine ut pudore contineatur. Hinc ubi, præ claritate, principia directa deſunt, ſufficit probatio per

per Enthymema. Quippe „ omnis de-
„ monſtratio ſupponit obſcuritatem quan-
„ dam, nec lumen ullum alio lumine
„ clarius reddi poteſt."

6. Nunc finge, quod valde poſſibile eſt, Catenam propoſitam repreſentare univerſaliter & in genere concatenationes quaſcumque imaginabiles, & quidquid de annulis illius oſtendimus conveniet concatenationi cuicumque in individuo determinatæ: quia *quidquid prædicatur de genere, verum eſt de omnibus individuis in quibus idea generis continetur*. Experturus, in charta ſeorſim ſcribe, ſub eadem linea, tres ideas quibus conſtant ſinguli ſillogiſmi ut & ſophiſmata propoſita in Logicis vulgatis, ita ut *media* inter extremas ponatur; & ſola inſpectione patebit regulam noſtram ſimplicem (3) perpetuo obtinere: præterquam quod mens, eo exercitio, habitum recte ratiocinandi, mira eum facilitate, acquirat.

7. Materia ſillogiſmorum in Algebra ſunt ideæ quantitatum abſtractæ & univerſales, quarum aliæ *datæ*, aliæ *quæſitæ*. Idea abſtracta eſt quæ in alia idea continetur; talis eſt, idea media in omni ſillogiſmo (3). Idea Univerſalis eſt idea abſtracta in multis ideis contenta; ut idea quantitatis quæ continetur in idea

omnis numeri aſſignabilis & determinati. Hac univerſalitate idearum fit ut enunciata fere omnia in *Analiſi* & præſertim concluſiones, *Theoremata* dici poſſint; quatenus omnis concluſio naturam præmiſſarum ſequi debet.

Verum mens noſtra ſine ſignis vix poteſt negotiari, nec numeri, ut pote determinati ſinguli poterant eſſe ſigna diſtincta idearum univerſalium. Hinc pro deſignandis ideis univerſalibus uſu-venerunt litteræ ſimplices, initiales *a b c d* &c. pro *datis* & finales *z y x* &c. pro *quæſitis*. His ſignis ſat inter ſe diſtinguuntur pauca *data* & *quæſita*, quæ in ſingulis problamatibus reſolvendis ſimul conferri debent tamquam annuli catenæ (2): nec opus erat prolixioribus ſignis obruere ſignatum.

8. Hactenus recte. Sunt ideæ ſigna naturalia rerum, communia græco & barbaro, voces vero ſigna mere arbitraria idearum. Verum opportet ideas concatenari, ut fiant propoſitiones ſeu æquationes. Adeoque, præter quantitatem *in abſtracto*, ſtatus illius *affirmativus* vel *negativus* conſiderandus veniebat. Nam quantitates, vel affirmativæ ſunt ſeu *nihilo majores*, vel negativæ ſeu *nihilo minores*; hoc eſt, *affirmativæ* ſunt exceſſus reales ſupra

pra o & *negativæ* defectus reales infra o; ut &c. $-4, -3, -2, -1, 0 +1 +2 +3 +4$ &c. Hinc ſit ut Æquatio ſit congeries quantitatum, ſeu *Totum cujus partes ſimul nihilo ſunt æquales.* (*Arith. univ. pag.* 52.) Quod impoſſibile foret, niſi partes *totius* aliæ nihilo majores & aliæ nihilo minores eſſent.

9. Arduum non erat quantitates litteris a, b, .. x, y &c deſignare & ſingularum ſtatum poſitivum ſigno $+$ quod eſt *plus*, negativum vero ſigno $-$ quod eſt *minus* exprimere, ut & *æqualitatem* ſigno $=$, ſed an, his poſitis, æque facile eſt exercere quatuor operationes in Analiſi ac in Arithmetica vulgari? Difficultas major videtur in Multiplicatione & Diviſione; ſed cùm in utraque diſciplina $+1$ ſit omnis multiplicationis terminus *primus* & omnis diviſionis *tertius*, hac ſola obſervatione mox innoteſcit quale ſignum, $+$ an $-$, ſit Producto vel Quoto cuicumque præfigendum.

10. Nunc *Theoriam* traditam (2. & ſeqq.) applicabimus doctrinæ *Combinationum* quæ in compoſitionem omnium æquationum influit & paſſim omittitur ab Analiſtis. Quippe datur in ſingulis, ut notum eſt, ſumma & numerus radicum, ſumma & numerus *factorum* ſeu productorum

ſub ſingulis radicibus binis, ternis, quaternis &c. Hinc

Problema. „ Dato numero m rerum „ (7) invenire numerum combinationum „ poſſibilium ſub ſingulis binis, ternis, „ quaternis &c"?

Reſp.. Numerus combinationum erit...

ſub ſingulis binis $m \times \frac{m-1}{2}$

ſub ſingulis ternis $m \times \frac{m-1}{2} \times \frac{m-2}{3}$

ſub ſing. quaternis $m \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$,

Ita in infinitum. Sed nota crucem x indicare multiplicationem quantitatum quibus interjacet.

Demonstratio. 1°. Combinando binas res, 1ª conjungitur omnibus, dempta una ſcilicet ſeipſa; 2ª, 3ª, 4ª, &c ſimiliter.

Sed (*ex hypotheſi*) 1ª. 2ª, 3ª, 4ª. &c, numerus eſt m & omnium, dempta una, numerus $m-1$.

Ergo combinando binas numerus rerum conjunctarum eſt $m \times (m-1)$.

Verum, combinando binas, numerus rerum conjunctarum eſt duplus numeri combinationum.

Ergo numerus combinationum eſt $m \times \frac{m-1}{2}$. Q. E. D. 1°.

(2°)

(2°) Combinando ternas, combinatio 1ª ſub binis ſemel conjungitur omnibus rebus, demptis duabus quas jam habet, 2ª, 3ª, 4ª, &c ſimiliter.

Sed 1ª, 2ª, 3ª, 4ª, &c combinationum ſub binis numerus eſt $m \times \frac{m-1}{2}$. & omnium rerum, demptis duabus, numerus eſt $m-2$.

Ergo combinando ternas, numerus rerum conjunctarum eſt $m \times \frac{m-1}{2} \times (m-2.)$

Verum combinando ternas, numerus rerum conjunctarum eſt triplus numeri combinationum.

Ergo numerus combinationum eſt $m \times \frac{m-1}{2} \times \frac{m-2}{3}$. Q. E. D. 2°. Cætera per analogiam innoteſcunt.

Examen. Sint a, b, c, d, res quatuor, erunt combinationes . . .

ſub binis *ab*.	ſub ternis *abc*.	ſub 4ternis *abcd*.
ac	*abd*	1
ad		
bc	*acd*	
bd		
cd	*bcd*	

Combin. $6 = m \times \frac{m-1}{2}$. $4 = 6 \times \frac{m-2}{3}$ & $1 = 4 \times \frac{m-3}{4}$. *Ut apparet..*

II.

11. Omnis ſillogiſmus reduci poteſt in propoſitionem *cauſalem* veram, & viceverſa, ut hæc „ *annulus* 1*us* ſuſtentat *ultimum*, „ quia ſuſtentat 2*um*, qui ſuſtentat 3*um* „ &c. „ Et hæc” numerus combinationum ſub binis eſt $m \times \frac{m-1}{2}$, quia *m* numerus rerum, multiplicatus per *m*−1, „ eſt duplus numeri combinationum.” At reduci nequit in ſillogiſmum ſequens cauſalis: „ Regula Newtoni non detegit interdum radices omnes impoſſibiles, „ quia in æquationibus numerus radicum „ excedit ſemper unitate numerum collationum poſſibilium.” Quin ita *retorqueri* poteſt argumentum „ Allegata cauſa „ vel obſtat ſemper, vel nunquam; ſed „ non obſtat ſemper: ergo obſtat nunquam.” Hinc delenda §. 114. apud Cel. *Maclaurin*, *Traité d'Algebre part.* 2. *ſect.* 1.

12. Sillogiſmus *conditionalis* non eſt niſi Enthymema duplex, in quo nempe non invenias ideam mediam continentem & contentam (3). An hoc eſt ſillogiſmus „ Si *annulus* 1*us* ſuſtentat 2*um* ſuſtentat „ 3*um*: ſed 1*us* ſuſtentat 2*um*: ergo & „ 3*um*?” An in eo datur major & minor propoſitio? (3)

13. Atque hactenus expoſita principia ſufficiunt iis quibus Logica definitur *Ars*

dete-

detegendi verum; mihi vero, cui animus eſt fines ejus proferre, deffinietur *Ars detegendi verum via breviori.* Ecquis audiat ſine nauſea hominem demonſtrantem viginti propoſitionibus id quod uno ſillogismo demonſtrari poteſt? Sicut a puncto ad punctum duci poſſunt infinitæ lineæ curvæ & una duntaxat recta: ita in aſſequendo vero, Mens varias curvas ingredi valet & unam tantùm rectam, quæ via ſemper eo clarior quo brevior eſt. Exemplum eſto rurſus Catena directe pendula (2) cujus annulus ultimus ſupponatur inſuper a primo ſuſtentari, mediantibus catenis multis orbiculariter trahentibus: An rite demonſtrat & an in modo non peccat, qui omiſſa catena media, annulos orbiculariter diſpoſitos percurrit? Immane quantum *viæ brevioris* neglectu voluminum excreverit moles, quantaque multis, qui mente mea præſtantiori pollent, redactionis in omnibus ſcientiis aperiatur meſſis. Id quod præſtitit in Inſtitutis *Juſtinianus*, in re medica *Boerhavius*, nuſpiam factum in aliorum ſcripta commentando, ſed potius ſpinas obvias averruncando. Si Logica eſt *Ars detegendi verum* præciſe, dic cur hodie non logica ſed mathematica *Methodus* appellatur? Cur Scri-

ptorum

ptorum etiam Principum in Analiſi, ſi per regulam *de breviori via* ſequenda judicentur, nullus abſolutus abeat? Aſſertio quidem gravis, ſed mox exemplis probatur. Eſto . . .

Prob. III. *Arith. Univ. pag.* 68. (quod primo loco pono, quia ex hoc pendet primi ſolutio). *Quantitatem datam ita in partes quotcumque dividere ut majores ſuperent minimam per differentias datas?*

Logice. „ ſubduc de quantitate data
„ differentias datas, reſtabit pars mi-
„ nima toties, quoties unitas eſt in nu-
„ mero partium „ Ergo reſiduum divi-
„ ſum per numerum partium erit pars
„ minima". Unde dantur reliquæ. Q. E. I.

Exemplum. Eſto linea 20 pedum ſic ſecanda in quatuor partes, ut ſupra primam ſit exceſſus 2*dæ* pedum 2, 3*tiæ*, ped. 3, & 4*tæ* ped. 7. erit pars minima $\frac{20-12}{4}=2$. *Ut opport.*

In caſu vero quo *pars maxima ſuperat minores per differentias datas*, „ Adde
„ quantitati datæ differentias datas, erit
„ aggregatum pars maxima toties, quo-
„ ties unitas eſt in numero partium: Er-
„ go aggregatum diviſum per numerum
„ partium erit pars maxima." Q. E. I.
Hinc . . .

14. THEOREMA I. Eſto $2a$ totum biſectum, $2d$ differentia partium, x pars major & y pars minor (7) eritque $x = a + d$ & $y = a - d$. nam $2x = 2a + 2d$ & $2y = 2a - 2d$.

Sed $xy = a^2 - d^2$. (25) Ergo $d^2 = a^2 - xy$ & $d = \sqrt{a^2 - xy}$. Hinc

15. *Theorema* II. „ Semidifferentia partium ſummæ cujuscumque biſectæ eſt „ radix quadrati ſemiſummæ, dempto „ producto partium". Idem infertur ex „ lib. Elem. II. *prop.* 5.

Hoc & præc. Theoremate reſolvitur omnis Æquatio 2di gradus, in qua (ut notum eſſe debet) „ quæruntur duæ quantitates „ quarum datur ſumma & productum." Ut ſi ſumma 16 & productum 55, erit major $8 + \sqrt{64 - 55} = 11$, & minor $8 - \sqrt{64 - 55} = 5$. *ut opport.* Nam $5 + 11 = 16$ & $5 \times 11 = 55$.

Coroll. 1. „ Factus ex Binomio in Apotome eſt differentia quadratorum partium". nam (*Theor.* 1.) $(a + d) \times (a - d) = a^2 - d^2$. Sat patet quid nobis ſit *Apotome*.

Corol. 2. „ Poſſibile eſt productum duarum partium vel *minus* vel *æquale*" „ quadrato ſemiſummæ, impoſſibile vero

„ *ma-*

„ *majus.*" Oriuntur ex tertio caſu, radices *impoſſibiles*, quæ ergo nunquam niſi numero pari in æquationibus compoſitis occurrunt; ex primo, radices reales *ſurdæ*, tantum non omnes, &c. & ex ſecundo, poteſtates perfectæ, ſimul & methodus nova & brevior extrahendi radices trinomias quadratas, ordinando terminos aliter ac fieri ſolet (17. 28.)

Ita eſto $x^4+2ax^3+a^2x^2+4b^2x^2+4b^2ax+4b^4=0$

Ordinando $x^2a^2\pm 2x^3a\pm 4b^2xa\pm$ &c. $=0$.

Fit divid. $a^2\pm 2xa\pm 4b^2a:x+$ &c. $=0$

Ergo(Th.I.II.) $a\pm x\pm 2b^2:x+0=0$

Et . . . . . $ax\pm x^2\pm 2b^2=0$. *rad. quæſita.*

Ad hæc confer Cel. CLAIRAUT *Traité d'Algebre* a *pag.* 111. ad 131. Quod *ibid. pag.* 136. reſolvitur paginis tribus problema pro inveniendis tribus quantitatibus geometrice proportionalibus, quarum datur ſumma & ſumma quadratorum, nos per ideas medias duas, quas omittunt omnes, longe clarius duplici tantummodo Enthymemate reſolvimus (43. *Prob.* XIII.)

PROB. I. (*Arith. pag.* 67.) *Invenire duas quantitates x & y, quarum datur ſumma a & differentia b quadratorum?*

(Per Theor. II. Cor. I.) $x-y=\frac{b}{a}$. Ergo

(Th.I.) $x=\frac{1}{2}a+\frac{b}{2a}$ & $y=\frac{1}{2}a-\frac{b}{2a}$. Q. E. I.

EXEMPL.

EXEMPL. Eſto $a=8$ & $b=16$. erit x ſeu $y=4\pm 1$. *Ut Opport.*

PROB. II. *Arith. Univ. pag.* 67. *Invenire tres quantitates* x, y & z, *quarum paris cujusque ſumma datur?*

Sit $x+y=a$ & $x+z=b$ & $y+z=c$, erit $2x+2y+2z=a+b+c$, quod ut pote cognitum dic $2d$. Ergo $d-a=z$ & $d-b=y$ & $d-c=x$. Q. E. I.

EXEMPL. Eſto $a=9$, $b=10$, & $c=13$. erit $x=3$. $y=6$ & $z=7$. *Ut Opport.*

Hoc ipſum reſolvi ſolet tribus variis modis, 1° propoſitionibus novemdecim, 2° & 3° propoſitionibus decem & quatuor. Confer MACLAURIN *Algebr.. part.* 1. *ſect.* 3. Quamvis ejusmodi reſolvi poſſint in infinitum uno Enthymemate (per 10.) Nam ſicut in combinatione trium litterarum ſub binis, ſingulæ bis occurrunt, ita in combinatione quatuor ſub ternis, ſingulæ ter &c. Confer Cl. s' GRAVESAND. *Matheſ. Univ. Elem. pag.* 63. 64. 65.

Quin etiam uno Enthymemate reſolves id quod MACLAURIN *ſect. ead.* 3. *cit.* propoſitionibus 20; nempe, $x+y+z=12$. $x+2y+3z=20$. & $\frac{1}{3}x+\frac{1}{2}y+z=6$ Nam de 2da æquatione ſubduc 3^{tiam} triplicatam & reſtabit $\frac{1}{2}y=2$ ſeu $y=4$.

Eadem *ſect.* 3. operoſius, propoſitionibus 12, quæruntur duæ quantitates *x* & *y* in proportione ut 2 ad 3, quarum ſumma ſit $\frac{1}{6}$ producti. Nam (per *Arithmetic.*) $y=\frac{3}{2}x$ & (ex *hypotheſ.*) $x+\frac{3}{2}x=\frac{1}{6}xy$. Ergo $1+\frac{3}{2}=\frac{1}{6}y$ & $y=15$. Q. E. I.

Ibid. part. 2. *ſect.* 1. Demonſtratur, paginis novem in 4°, Newtoni regula inveniendi ſummas poteſtatum radicum æquationis cujuſcumque; quam ope novæ ideæ mediæ, duplici tantum Enthymemate, directe & longe clarius demonſtrabimus (34 *Theor.* 1).

15. Nec me Cordatus quiſquam ægre ferret ſummorum virorum redarguerem, dum in ſcientiis tradendis non minus nocet nimia quam nulla prolixitas; quippe ſi propoſitio quævis per adjectam vel unam vocem ſuperfluam, obſcuritatem nonnullam contrahat, quidni & per propoſitiones inutiles Concluſio quæcumque (2 & 3)? Cæterumquin judicium eſto penes Lectorem æquum, an in opere ſequenti vel latum unguem diſceſſerim a Canone mihi in frænum aſſumpto, qui & Utinam, pro felicitate rei litterariæ, in omnibus ſcientiis obtineret!

CANON.

Quidquid unus omnium optime dixit, non aliter dicere, niſi melius dicatur.

ARITHMETICA UNIVERSALIS.

POSTULATA.

Liceat I. Quantitates per litteras exprimere, *cognitas* quidem per initiales *a*, *b*, *c*, *d*, &c, *incognitas* vero per finales *z*, *y*, *x*, *v* &c. (7)

II. Quantitates *affirmativas* ſeu nihilo majores notare, præfixo ſigno + quod ſignificat *plus*, *negativas* autem ſeu nihilo minores, præfixo ſigno − quod ſignificat *minus* & ubi neutrum initiali quantitati præfigitur ſignum + ſub-intelligere. (8)

III. Quantitates connexas ſignis propriis pro additis, ſignis vero mutatis, pro ſubductis habere. ſic *a*−*b* ſignificat defectum −*b* additum exceſſui +*a* ſeu exceſſum +*b* ſubductum de +*a*. (9)

IV. Duas vel plures litteras immediate vel mediante ſigno x connexas, habere pro *facto* ſeu quantitate quæ fit per multiplicationem omnium in ſe invicem, ut *ab* x *cd* pro *abcd*. Et $\frac{ab}{cd}$ vel *ab* : *cd* pro *quoto*

ex diviſione ab per cd. Imo habere numerum immediate litteræ *præfixum* pro coefficiente totius, ut $2a$; *ſuffixum* pro exponente poteſtatis, ut a^2, a^3; *infixum* ſigno $\surd$ pro radice poteſtatis ut $\sqrt[2]{a}$ ſeu $\surd a$, $\sqrt[3]{a}$, &c

V. *Æqualitatem* notare ſigno $=$, *majoritatem* ſigno $>$ anguli vertice minorem partem indicante; *proportionem* ſigno $\div$ vel $::$ vel ut $=$ inter puncta $:$; nam ſi $a : b :: c : x$ erit $\frac{b}{a} = \frac{x}{c}$ & $\frac{bc}{a} = x$ & $bc = ax$ (*Vid.* 18. *Axiom.* III. & IV.

17. DEFINITIONES.

I. Æquatio eſt totum cujus partes ſimul ſunt æquales nihilo: ut $x = a$ ſeu $x - a = 0$. (8)

II. Æquatio *ſimplex* eſt cujus pars incognita non excedit poteſtatem primam, quidquid ſit de cognita. Ut $x - a^2 = 0$.

III. Æquatio *compoſita* eſt cujus incognitæ exponens unitatem excedit, ſeu eſt factus duarum vel plurium æquationum ſimplicium (18. *ax.* IV.) ut $x^2 - a^2 = (x + a) \times (x - a) = 0$.

IV. *Radix* æquationis eſt pars factoris, quæ ſubſtituta pro incognita facit omnes ter-

terminos evaneſcere: ut $+a$ & $-a$ reſpectu facto ad $x^2-a^2=0$.

V. Æquatio *numerica* eſt cujus radices omnes ſunt numeri: ut $x^2-9=0=(x+3)\times(x-3)$.

VI. Æquatio *litteralis* eſt cujus radices omnes ſunt litteræ, vel cujus termini ſinguli ad dimenſiones æque altas aſcendunt. Ut $x^2-2ax+a^2=0$. Hinc patet quid ſit æquatio *mixta*.

VII. Æquatio *ordinata* eſt in cujus terminis Exponentes incognitæ ſequuntur progreſſionem Arithmeticam, ut $x^3+px^2+qx^1+rx^0=0$. ſeu (ob $x^0=1$) $x^3+px^2+qx+r=0$. Nec obſtat ſi deſint interdum termini ex intermediis, quorum loca ſolent aſtherismo * notari.

VIII. Æquatio *pura* eſt, cujus ordinatæ termini primi coefficiens eſt unitas, & quæ nullam quantitatem radicalem in terminis monſtrat.

IX. Æquatio *affecta* eſt, cujus radices una vel plures ſunt fractiones, ut $6x^2-5x+1=(x-\frac{1}{2})\times(x-\frac{1}{3})=0$, vel cujus termini radicale ſignum monſtrant.

X. Poteſtas *perfecta* vocatur æquatio cujus factores nihilo differunt; ut $x^2+2ax+a^2=(x+a)\times(x+a)$.

18. AXIOMATA.

I. Æqualia uni tertio ſunt æqualia inter ſe.

II. Manet æqualitas ſive addas ſive ſubducas æqualia æqualibus.

III. Manet æqualitas ſive multiplices ſive dividas æqualia per æqualia.

IV. Nil magis naturale eſt quam unumquodque eodem modo reſolvi quo colligatum eſt.

Hinc in æquationum reſolutione, quæ obſcuritas oritur ab additione tollatur per ſubductionem & quæ à multiplicatione per diviſionem, nec non viceverſa.

V. Si totum fuerit in duas partes ſectum, addaturque toti differentia partium, fiet pars minor æqualis majori & ſi de toto ſubducatur differentia partium, fiet pars major æqualis minori.

ADDITIO.

19. ADDITIO „ fit connectendo quanti-
„ tates addendas cum ſignis propriis &
„ inſuper uniendo quæ poſſunt uniri."
Sic a & $b = a + b$ & a & $- b = a - b$
& $- a$ & $- b = - a - b$ vel $- b - a$.
„ Poſſunt uniri quæ ex parte litterali

„ con-

,, conveniunt." Sic $7a + 9a = 16a$; $6\sqrt{ab+xx} + 7\sqrt{ab+xx} = 13\sqrt{ab+xx}$. Quin & $a\sqrt{ac} + b\sqrt{ac} = (a+b)\sqrt{ac}$. Imo $+\frac{11ax}{b} - \frac{4ax}{b} = \frac{7ax}{b}$. & $-a\sqrt{ac} + b\sqrt{ac} = (b-a)\sqrt{ac}$. & $2\sqrt{ac} - 5\sqrt{ac} = -3\sqrt{ac}$.

In Additione magis compoſita terminos ſequenti modo ordinare convenit.

pars . . .	$4a + 2b - 2c + 5d - 9$
pars . . .	$5a \quad\quad + 6c - 8d - 39$
Add. . . .	
Totum . . .	$9a + 2b + 4c - 3d - 48$.

SUBDUCTIO.

26. SUBDUCTIO ,, fit connectendo quantitates cum ſignis omnibus quantitatis ,, ſubducendæ mutatis & inſuper uniendo quæ poſſunt uniri" Sic $7a$ de $9a = 9a - 7a = 2a$. & $-7a$ de $9a = 9a + 7a = 16a$. & $-9a$ de $7a = 7a + 9a = 16a$ & $-9a$ de $-7a = 9a - 7a = 2a$. Imo $(a-x)\sqrt{ax}$ de $(a+x)\sqrt{ax} = (a+x-a+x)\sqrt{ax} = 2x\sqrt{ax}$. Ubi agitur de Polinomiis convenit ordinare terminos ſequenti modo,

Totum	$9a + 2b + 4c - 3d - 48$
pars	$5a \quad\quad + 6c - 8d - 39$
ſubd.	

pars reſidua . . $4a + 2b - 2c + 5d - 9$

Rurſus *Totum* . . . . $\frac{11ax}{b} - 7\sqrt{3} + \frac{2}{3}$

pars . . . . $\frac{4ax}{b} - 6\sqrt{3} - \frac{1}{5}$

ſubd. . . . . . ————————————

pars reſid. . . . . $\frac{7ax}{b} - \sqrt{3} + \frac{3}{5}$

MULTIPLICATIO.

21. In omni multiplicatione „ quæritur quarta proportionalis, quæ diviſa „ per tertiam, æquetur ſecundæ diviſæ „ per + 1 " Hinc tres regulæ veniunt hic obſervandæ, ſcilicet *ſignorum*, *Coefficientium* & *litterarum*.

Regula ſignorum (9).

Multiplicando + per + vel - per - fit +, non -.

Nam + 1 : + = + : +

& + 1 : - = - : +

ſed multiplic. + per -, vel - per + fit -, non +.

Nam + 1 : + = - : -.

& + 1 : - = + : -.

Regula Coefficientium.

22. „ Si coefficientes ſint, numerales „ duc coefficientes unius factoris in coef- „ ficien-

„ ficientes alterius” ut fit in multipli-
catione Arithmetica.

23. *Regula Litterarum.*

„ Scribe litteras unam poſt alteram,
„ quo ordine nil refert”. Sic $2a \times 3b = 6ab$ vel $6ba$. & $-2a \times -3b = 6ab$. Item $-2a \times 3b$, vel $2a \times -3b = -6ab$. & $-4z \times -3\sqrt{az} = 12z\sqrt{az}$. Porro $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

& $2\frac{a}{b} \times 3\frac{c}{d} = 6\frac{ac}{bd}$ ſeu $6a\frac{c}{bd}$ &c.

24. „ Quantitates radicales, hoc eſt
„ ambæ radices quadraticæ vel ambæ
„ cubicæ &c. multiplicantur ducendo
„ terminos in ſe invicem ſub eodem ſigno
„ radicali.” Sic $\sqrt{ab} \times \sqrt{cd} = \sqrt{abcd}$. &
$\sqrt{\frac{a^3}{c}} \times \sqrt{\frac{abb}{c}} = \sqrt{\frac{a^4bb}{cc}}$ hoc eſt $\frac{aab}{c}$, nam $\sqrt{a^4} = aa$, $\sqrt{bb} = b$ & $\sqrt{cc} = c$. Et $2a\sqrt{az} \times 3b\sqrt{az} = 6ab\sqrt{a^2z^2} = a^2bz$.

Nec major fere difficultas eſt multiplicandi polinomium per polinomium: cum operatio non fiat niſi per monomia ſigillatim, quæ ſingula lineam unam facti totalis abſolvunt, atque ſi monomium multiplicans ſit ſub ſigno +, ſigna hujus lineæ ſequentur ſigna polinomii mul-

 tipli-

tiplicandi; ſecus ſi fuerit ſub ſigno −.

EXEMPLA.

Factor	$a^2 + 2ac - bc$	$a^2 + ab + b^2$
Factor	$a - b$	$a - b$
×		
Lin. 1ª ...	$a^3 + 2a^2c - abc$	$a^3 + a^2b + ab^2$
Lin. 2ª ...	$- a^2b - 2abc + b^2c$	$- a^2b - ab^2 - b^3$
Add. ...		
Factus	$a^3 - a^2b + 2a^2c - 3abc + b^2c$	$a^3 \quad * \quad * \quad b^3$

Porro.	$d = a + b$	$a + b$	$\sqrt{a} + \sqrt{b}$
	$d = a + b$	$a - b$	$\sqrt{a} - \sqrt{b}$
× ...			
	$a^2 + ab$	$a^2 + ab$	$\sqrt{a^2} + \sqrt{ab}$
	$+ ab + b^2$	$- ab - b^2$	$- \sqrt{ab} - \sqrt{b^2}$
Add. ..			
Factus	$d^2 = a^2 + 2ab + b^2$	$a^2 \; * \; - b^2$	$a \; - \; b$

25. THEOREMATA tria docet ultima hæc linea. 1um „ Quadratum binomii, „ præter quadrata partium, continet „ duo rectangula ex partibus", 2^{dum} „ Factus *Binomii* in *Apotome* eſt differentia „ quadratorum partium" 3^{tium} „ Nulla „ datur æquatio abſolute ſimplex. 4°" „ Si de quadrato totius in partes quotcumque ſecti ſubducas duplam ſum- „ mam

„ mam factorum ſub ſingulis binis (9) „ reſtabit ſumma quadratorum partium." Ex primo, quod eſt Euclid. II. prop. 4 deducitur 4um.

Rurſus. Factor ... $\frac{2ax}{c} - \sqrt{\frac{a^3}{c}}$

Factor. $3a + \sqrt{\frac{abb}{c}}$

I ......

$\frac{6a^2x}{c} - 3a\sqrt{\frac{a^3}{c}}$

$+ \frac{2ax}{c}\sqrt{\frac{abb}{c}} - \sqrt{\frac{a^4b^2}{cc}}$

Add. ....

Factus ... $\frac{6a^2x}{c} - 3a\sqrt{\frac{a^3}{c}} + \frac{2ax}{c}\sqrt{\frac{abb}{c}} - \frac{a^2b}{c}$

DIVISIO.

26. Diviſio „ fit reſolvendo quid- „ quid per multiplicationem conflatur", In ea tres regulæ ſequentes obſervantur.

Regula ſignorum (21).

„ Si ſigna diviſoris & dividendi fue- „ rint ſimilia, ſcribe + in quoto, ſi diſ- „ ſimilia −". Nam + 1 eſt omnis diviſionis terminus tertius.

Re-

Regula coefficientium.

„ Divide coefficientes dividendi per „ coefficientes diviſoris" ut fit in Arithmetica.

Regula litterarum.

„ Tolle de dividendo litteras quæ ſunt „ in diviſore, nec niſi reliquas in quoto „ ſcribe" ſed ſi diviſio totius abſolvi nequit, ſubſcribatur reſiduo diviſor, ut fit in Arithmetica.

Sic $\frac{ab}{a}$ ſeu $ab : a = b$. $\frac{a^2}{a} = a$. $\frac{ab^2}{b} = ab$.

$\frac{a}{a} = 1$. $\frac{a}{\sqrt{a}} = \sqrt{\frac{aa}{a}} = \sqrt{a}$. & $(a^2 + 2ax - x^2) : a = a + 2x - \frac{xx}{a}$.

27. At ubi diviſor eſt polinomium, ordinatis terminis (17. D. 7.) diviſio perinde ac in numeris inſtitui debet: „ ſcilicet, „ divide ſucceſſive terminos dividendi „ per primum terminum diviſoris; duc „ totum diviſorem in terminum quoti „ ſic inventum; factum hocce ſubduc „ de dividendo. Eodem modo opus „ iteretur ſuper reſiduo dividendo, quo„ usque, ſi fieri poteſt, nihil ex divi„ dendo ſuperſit" ut in ſequenti Diagrammate.

4a +5b)

$$4a+5b)\ 8a^2-2ab-15b^2\ (2a-3b$$
$$\underline{8a^2+10ab}$$
$$0\ -\ 12ab-15b^2$$
$$\underline{-\ 12ab-15b^2}$$
$$0$$

In quo, linea prima repreſentat diviſorem ſiniſtrorſum; dividendum inter arcus & quotum ſiniſtrorſum: ſecunda factum ex $2a$ in $(4a+5b)$ ſubducendum; tertia, reſiduum a ſubductione; quarta, factum ex $-3b$ in $(4a+5b)$ ſubducendum & quinta, nihil; quia reſiduum dividendi eſt o.

Idem prodiret quotus, ſi diviſor & dividendum ordinarentur juxta dimenſiones litteræ b, ut ſequitur.

$$5b+4a)\ -15b^2-2ab+8a^2\ (-3b+2a$$
$$\underline{-15b^2-12ab}$$
$$0\ +10ab+8a^2$$
$$\underline{+10ab+8a^2}$$
$$0$$

Examen $(5b+4a)\times(2a-3b)=8a^2-2ab-15b^2$.

Id applicari poteſt omnibus exemplis ſupra in multiplicatione traditis, aſſumendo pro diviſore factorem unum vel alterum. Si horum unus datur fat facilis eſt diviſio, ſed ubi nullus, factores a ſe invicem ſecernere, hoc eſt quod ſolet eſſe difficillimum & idem quod æquationes re-

reſolvere, ubi factores polinomii nihilo ſupponuntur æquales. Si factores ſimplices terminum, ſaltem unum, communem habent reſolvi quidem poteſt (per 13.) omnis æquatio ſecundi gradus, non ita tertii, quarti, nec pro quinto & altioribus gradibus ulla ſpes reſolutionis exactæ & univerſalis ſupereſt. Verum, ubi factores non differunt, locum habet reſolutio per *Extractionem radicis.*

EXTRACTIO RADICUM.

28. E monomiis extractio radicum ex ipſa notatione patet, ut $\sqrt{aa} = a$. $\sqrt{\frac{a^4}{cc}}$ ſeu $\sqrt{a^4} : \sqrt{cc} = \frac{a^2}{c}$ & $3c\sqrt{\frac{9aazz}{25bb}} = 3c \times \frac{3az}{5b}$ ſeu $\frac{9acz}{5b}$. Sed ubi agitur de polinomiis opus perinde ac in numeris abſolvitur. Sic extracturus radicem quadraticam ex $a^2 + 2ab + b^2$, radicem a quadrati a^2 in quoto ſcribe & ablato ejus quadrato a^2 reſtabit $2ab + b^2$. Dic itaque (25) quoties duplum quoti ſeu $2a$ continetur in primo reſiduo

$$
\begin{array}{r|l}
a^2 + 2ab + b^2 & (a + b \\
a^2 & \\
\hline
2a)\ 0 + 2ab + b^2 & \\
+ 2ab + b^2 & \\
\hline
0 &
\end{array}
$$

$2ab$? *Reſp.* b. Hinc in quoto ſcribe b & ablato b ducto in $(2a+b)$ ſeu $2ab+b^2$, reſtabit 0; indicio radicem eſſe $a+b$.

Pari modo invenietur x^2+2x-2, radix quadrata ſequentis polinomii.

$$x^4+4x^3 * -8x+4\,(x^2+2x-2.$$
$$x^4.$$
$$:2x^2)\;\ldots 0\;+$$
$$\text{ſubd.}\;\ldots\; 4x^3+4x^2.$$
$$:2x^2+4x)\;0\;-4x^2-$$
$$-4x^2-8x+4.$$
$$0\quad 0\quad 0.$$

(15 *Carol.* 2.)

Si radicem cubicam ex $a^3+3a^2b+3ab^2+b^3$ oportet extrahere, operatio eſt hujuſmodi.

$$a^3+3a^2b+3ab^2+b^3\,(a+b$$
$$a^3$$
$$:3a^2)\;\ldots\; 0+3a^2b\,(b$$
$$\&\;\ldots\;(a+b)^3=a^3+3a^2b+3ab^2+b^3 \text{ quo}$$
$$\text{ſubducto reſtat}\;\ldots\; 0\quad 0\quad 0\quad 0$$

Eadem modo radix cubica, ſi extrahatur ex $x^6+6x^5-40x^3+96x-64$, prodit x^2+2x-4. atque ita in altioribus radicibus.

REDUCTIO *fractionum & radicalium.*

29. Hæ reductiones fiunt vel ad *minimos terminos* vel ad *eandem denominationem*

&

& præcedentibus operationibus inſerviunt.

„ Fractiones ad *minimos terminos* reducuntur, dividendo numeratores ac denominatores per maximum communem diviſorem". Sic $\frac{a^2c}{bc}$ fit $\frac{a^2}{b}$ dividendo per *c*. & $\frac{6a^3-9acc}{6a^2+3ac}$ fit $\frac{2a-3cc}{2a+c}$ dividendo per $3a$. atque ita plerumque multiplicatio & diviſio poſſunt abbreviari; ſcilicet, diviſio reducitur in multiplicationem converſo tantum diviſore & in omni multiplicatione licet denominatores pro lubitu permutare: ut ſi multiplicare oportet $\frac{2ab^3}{3ccd}$ per $\frac{9acc}{bdd}$, vel id dividere per $\frac{bdd}{9acc}$, pone $\frac{2ab^3}{bdd} \times \frac{9acc}{3ccd} = \frac{2abb}{dd} \times \frac{3a}{d} = \frac{6aabb}{d^3}$. Atque ita $\frac{aa}{c} \times \frac{c}{b} = \frac{aa}{b} \times \frac{c}{c} = \frac{aa}{b}$. Et $\frac{aa}{c} : \frac{b}{c} = \frac{aa}{b} \times \frac{c}{c} = \frac{aa}{b}$. Et $\frac{a^3-ax^2}{xx} \times \frac{cx}{aa+ax} = \frac{a^3-ax^2}{aa+ax} \times \frac{cx}{xx} = \frac{a^2-x^2}{a+x} \times \frac{c}{x} = (a-x) \times \frac{c}{x} = \frac{ac}{x} - c$. (Vid. 18. *ax.* 3.)

„ Fractio

„ Fractiones reducuntur ad *communem* „ *denominatorem*, per (18 Ax. III. & „ IV)." Quod uſuvenit præcipue in Additione ut partes poſſint uniri, ſic $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$. Et $a + \frac{ab}{c} = \frac{ac}{c} + \frac{ab}{c} = \frac{ac+ab}{c}$. & $\frac{a^3}{bc} - \frac{a^3}{bd} = (bda^3 - bca^3) : bbcd = \frac{d-c}{bcd} a^3$. „ Radicalia ſæpe reducuntur ad *minimos terminos*, extrahendo radicem factoris alicujus." Sic $\sqrt{aabc} = a\sqrt{bc}$. & $\sqrt{48aabc} = 4a\sqrt{3bc}$, propter $48 = 3 \times 16$ & $a = \sqrt{aa}$. Sic $\sqrt{\frac{aaoomm}{ppzz} + \frac{4aam^3}{pzz}} = am\sqrt{\frac{oo}{ppzz} + \frac{4m}{pzz}} = \frac{am}{pz}\sqrt{oo+4mp}$.

Et $\sqrt[3]{8a^3b + 16a^4} = 2a\sqrt[3]{b+2a}$. Sic $\sqrt{\frac{4ab^3}{cc}} + \sqrt{\frac{a^3b - 4a^2b^2 + 4ab^3}{cc}} = \frac{2b}{c}\sqrt{ab} + \sqrt{\frac{aa-4ab+4bb}{cc}}\sqrt{ab} = \frac{2b}{c}\sqrt{ab} + \frac{a-2b}{c}\sqrt{ab} = \frac{a}{c}\sqrt{ab}$.

Radicalia *diverſæ denominationis* oppor-

tet ad eandem reduci, cum inſtituenda eſt multiplicatio vel diviſio, idque „ præ-
„ figendo ſignum radicale cujus index ſit
„ minimus numerus quem earum indi-
„ ces dividunt ſine reſiduo & ſuffixas
„ quantitates toties, dempta una vice,
„ in ſe ducendo quoties index ille jam
„ major evaſerit."

Sic enim $\sqrt[2]{ax} \times \sqrt[3]{a^2x} = \sqrt[6]{a^3x^3} \times \sqrt[6]{a^4x^2} = \sqrt[6]{a^7x^5}$. Et $\sqrt{a} \times \sqrt[4]{ax} = \sqrt[4]{aa} \times \sqrt[4]{ax} = \sqrt[4]{a^3x}$. Et $\sqrt{6} \times \sqrt[4]{\frac{5}{6}} = \sqrt[4]{36} \times \sqrt[4]{\frac{5}{6}} = \sqrt[4]{30}$. Porro $a\sqrt{bc} = \sqrt{aa} \times \sqrt{bc} = \sqrt{a^2bc}$. & $4a\sqrt{3bc} = \sqrt{16a^2} \times \sqrt{3bc} = \sqrt{48a^2bc}$. & $\sqrt{ac} : b = \sqrt{ac} : \sqrt{bb} = \sqrt{\frac{ac}{bb}}$. Et $\frac{6ab^2}{\sqrt{18ab^3}} = \sqrt{\frac{36a^2b^4}{18ab^3}} = \sqrt{2ab}$. Et ſic in aliis.

REDUCTIO RADICALIUM

Ad ſimpliciores radicales per extractionem radicum.

30. Radices quantitatum, quæ ex integris & radicalibus quadraticis componuntur, ſic extrahe.

„ De-

„ Deſignet A „ quadrati alicujus partem „ majorem, B partem minorem. Et erit „ $\frac{A+\sqrt{AA-BB}}{2}$ quadratum majoris „ partis radicis; & $\frac{A-\sqrt{AA-BB}}{2}$ „ quadratum partis minoris, quæ quidem majori annectenda eſt cum ſigno „ ipſius B."

Demonſtratio. Eſto $x^2+y^2=A$ & $2xy=B$ (25) erit $BB:4=xx\,yy$. Ergo (13) x^2 vel $y^2=(A\pm\sqrt{AA-BB}):2$. Q. E. D.

Hinc eſto $3+\sqrt{8}$ quadratum, $A=3$, $B=\sqrt{8}$. erit $\sqrt{AA-BB}=1$, indeque $x^2=\frac{3}{2}+\frac{1}{2}=2$ & $y^2=\frac{3}{2}-\frac{1}{2}=1$. Ergo $\sqrt{1}+\sqrt{2}$ ſeu $1+\sqrt{2}$. erit radix ex $3+\sqrt{8}$. Rurſus eſto $\sqrt{32}-\sqrt{24}$. quadratum; $\sqrt{32}=A$, $\sqrt{24}=B$. erit $\sqrt{AA-BB}=\sqrt{8}$. indeque $\frac{1}{2}\sqrt{32}+\frac{1}{2}\sqrt{8}$ & $\frac{1}{2}\sqrt{32}-\frac{1}{2}\sqrt{8}$, hoc eſt $3\sqrt{2}$ & $\sqrt{2}$ quadrata partium radicis. Radix itaque eſt $\sqrt[4]{18}-\sqrt[4]{2}$. Porro eſto $a^2+2x\sqrt{aa-xx}$ quadratum, $a^2=A$ $2x\sqrt{aa-xx}=B$. erit $AA-BB=a^4-4a^2x^2+4x^4$, cujus radix $aa-2xx$. Hinc $\frac{a^2+a^2-2x^2}{2}=a^2-x^2$ quadratum majoris

partis radicis & x^2 quadratum alterius, adeoque $x+\sqrt{aa-xx}$, eſt radix quæſita. Porro ſi habeatur $aa+5ax-2a\sqrt{ax+4xx}$, eſto $aa+5ax=A$ & $-2a\sqrt{ax+4xx}=B$, fiet $AA-BB=a^4+6a^3x+9a^2x^2$, cujus radix $aa+3ax$. Unde quadratum majoris partis radicis erit a^2+3ax, illud minoris ax & radix $\sqrt{aa+4ax}-\sqrt{ax}$. Denique quadrati $6+\sqrt{8}-\sqrt{12}-\sqrt{24}$, ſeu $6+2\sqrt{2}-2\sqrt{3}-2\sqrt{6}$, radix eſt trinomium, cujus ſumma quadratorum partium 6, adeoque ſumma factorum ſub ſingulis binis $\sqrt{2}-\sqrt{3}-\sqrt{6}$ ſeu $1\times\sqrt{2}+1\times-\sqrt{3}+\sqrt{2}\times-\sqrt{3}$: (25) Ergo $1+\sqrt{2}-\sqrt{3}$ erit radix quæſita.

31. Eſt & Regula extrahendi *altiores radices* ex quantitatibus numeralibus duarum potentia commenſurabilium partium:

„ Sit quantitas $A+B$. Ejus pars major A. Index radicis extrahendæ c. Quære minimum numerum n, cujus poteſtas n^c dividitur per $AA-BB$ ſine reſiduo & quotus ſit Q. Computa $\sqrt[c]{(A+B)\sqrt{Q}}$ in numeris integris proximis. Sit illud r. Divide $A\sqrt{Q}$ per maximum diviſorem rationalem: ſit quotus s fiatque $\frac{r+\frac{n}{r}}{2s}$ in numeris in-

„ te-

„ integris proximis r. Et erit $\frac{ts \pm \sqrt{ttss - n}}{\sqrt[2c]{Q}}$
„ radix quæſita, (ſi modo radix extrahi
„ poteſt."

DEMONSTRATIO regulæ Auctoris. $n^c = (AA - BB)\,Q = (A + B)\sqrt{Q} \times (A - B)\sqrt{Q}$ (25. Th. 2.)

Ergo $n = \sqrt[c]{(A + B)\sqrt{Q} \times (A - B)\sqrt{Q}}$ facto binomii in apotom.

Sed in numeris integris proximis, $r = \sqrt[c]{(A + B)\sqrt{Q}}$ ſummæ partium.

Ergo $\frac{r + \frac{n}{r}}{2} = \frac{1}{2}\sqrt[c]{(A + B)\sqrt{Q}} + \frac{1}{2}\sqrt[c]{(A - B)\sqrt{Q}}$ (14.)

Verum eſt $A\sqrt{Q}$ factus ex parte rationali & parte irrationali s, dividendo emungenda & poſt multiplicando reſtituenda. Hinc poſito $t = \frac{r + \frac{n}{r}}{2s}$ in numeris integr. prox. ſit rurſus $ts = (r + \frac{n}{r}):2$ in numerus integris proximis & (§. 13.) $ts \pm \sqrt{ttss - n}$ diviſa per $\sqrt[2c]{Q} = \sqrt[c]{\sqrt{Q}}$ erit radix quæſita, ſi modo radix extra-

hi poteſt: quippe hæc ſupponitur ex numeris integris conflari: hinc error determinando in numeris integris *proximis* nullus eſſe poteſt. Q. E. D.

Ut ſi ſit extrahenda radix cubica ex $\sqrt{968} + 25$, erit $AA - BB = 343$, ejus diviſores 7, 7, 7; ergo $n = 7$. & $Q = 1$. Porro $(A + B)\sqrt{Q}$ ſeu $\sqrt{968} + 25$, extracta prioris partis radice, fit paulo major quam 56; ejus radix cubica in numeris integris proximis eſt 4. Ergo $r = 4$. Inſuper $A\sqrt{Q}$ ſeu $\sqrt{968}$, extrahendo quidquid rationale eſt, fit $22\sqrt{2}$, ergo ejus pars radicalis $\sqrt{2} = s$, & $(r + \frac{n}{r}) : 2s = 5 : 2\sqrt{2}$ in numeris integris proximis 2. Ergo $t = 2$. Denique $ts = 2\sqrt{2}$ & $\sqrt{ttss - n} = 1$ & $\sqrt[2c]{Q}$ ſeu $\sqrt[6]{1} = 1$. Ergo $2\sqrt{2} + 1$. eſt radix tentanda. Tento itaque per multiplicationem ſi cubus ipſius $2\sqrt{2} + 1$ ſit $\sqrt{968} + 25$, & res ſuccedit.

Rurſus ſi radix cubica extrahenda ſit ex $68 - \sqrt{4374}$; erit $AA - BB = 250$, cujus diviſores 5, 5, 5, 2. Ergo $n = 5 \times 2 = 10$ & $Q = 4$ & $\sqrt[c]{(A + B)\sqrt{Q}}$ ſeu $\sqrt[6]{68 + \sqrt{4374} \times 2}$ in numeris integris proxi-

proximis eſt $7 = r$. Inſuper AVQ ſeu $68\sqrt{4}$, extrahendo quidquid rationale eſt, fit $136\sqrt{1}$. Ergo $s = 1$ & $(r + \frac{n}{r}) : 2s$ ſeu $(7, + \frac{10}{7}) : 2$ in numeris integr. prox, eſt $4 = t$. Ergo $ts = 4$ & $\sqrt{ttss - n} = \sqrt{6}$ & $\sqrt[2c]{Q} = \sqrt[6]{4} = \sqrt[3]{2}$. Hinc radix tentanda $(4 - \sqrt{6}) : \sqrt[3]{2}$.

Iterum ſi $\sqrt[5]{}$ ſeu radix quadratocubica extrahenda ſit ex $29\sqrt{6} + 41\sqrt{3}$; erit $AA - BB = 3$, adeoque $n = 3$, $Q = 81$, $r = 5$, $s = \sqrt{6}$, $t = 1$, $ts = \sqrt{6}$, $\sqrt{ttss - n} = \sqrt{3}$ & $\sqrt[2c]{Q} = \sqrt[10]{81}$ ſeu $\sqrt[5]{9}$ adeoque radix tentanda $(\sqrt{6} + \sqrt{3}) : \sqrt[5]{9}$.

Cæterum „ ſi quantitas fractio ſit, vel „ partes ejus communem habent diviſo„ rem, radices denominatoris & fa„ ctorum ſeorſim extrahe". Ut ſi ex $\sqrt{242} - 12\frac{1}{2}$ radix cubica ſit extrahenda; hoc reductis partibus ad communem denominatorem fiet $(\sqrt{968} - 25 : 2.)$ Dein, extracta ſeorſim numeratoris & denominatoris radice cubica, orietur $(2\sqrt{2} - 1) : \sqrt[3]{2}$. Rurſus ſi ex $\sqrt[3]{3993} + \sqrt[6]{17578125}$ radix aliqua ſit extrahenda, divide par-

tes per communem diviſorem $\sqrt[3]{3}$ & emerget quotus 11 + 125 ex quibus factoribus radices ſeorſim extrahe.

Non poteſt erui radix cubica $\frac{1}{2} + \frac{1}{2}\sqrt{5}$ quantitatis $2 + \sqrt{5}$ per formulam Auctoris, niſi $2 + \sqrt{5}$ ducatur in 8: quia ſupponit ille non niſi quantitates integras quæri.

Forma Æquationum.

32. Æquationes ſunt vel *mediæ* vel *finales* (2). *Mediæ* involvunt plures quantitates incognitas, quæ ideo debent conferri & ita connecti ut ex omnibus una tandem emergat æquatio *finalis* cui ineſt una tantum incognita quantitas, quam quærimus & quæ ſolis cognitis eſt involuta (18). Quæ ut inveniatur, variis plerumque modis transformanda eſt æquatio, donec ea evadat ſimpliciſſima quæ poteſt & ſimilis alicui ex ſequentibus earum gradibus, in quibus x deſignat quantitatem quæſitam, p coefficientem ſecundi termini, q tertii, r quarti, s quinti &c. (10), ex quibus p, q, r, s determinatis, x determinatur v. g.

$x = p$ vel $x - p = 0$.

$x^2 = px + q$ vel $x^2 - px - q = 0$

$x^3 = px^2 + qx + r$ vel $x^3 - px^2 - qx - r = 0$

$x^4 = px^3 + qx^2 + rx + s$, vel $x^4 - px^3 - qx^2 - rx - s = 0$

&c.

Quid

Quid autem inſuper intelligatur per p, q, r, s, t &c. ex ſola compoſitione patebit. (10. 21.)

$$\begin{array}{rl} & x-a=0 \\ \times\ldots\ldots & x-b=0 \\ \hline & x^2-ax \\ & \quad -bx+ab \\ \hline \end{array}$$

$$\text{ſeu}\ldots\; x^2 \begin{matrix}-a\\-b\end{matrix} x + ab = 0$$

$$\begin{array}{rl} \times\ldots\ldots\ldots & x - c = 0 \\ \hline & x^3-ax^2+abx \\ & \quad -bx^2 \\ & \quad -cx^2+acx-abc \\ & \qquad +bcx \\ \hline \end{array}$$

$$\text{ſeu}\ldots\; x^3 \begin{matrix}-a\\-b\\-c\end{matrix} x^2 \begin{matrix}+ab\\+ac\\+bc\end{matrix} x - abc = 0$$

$$\begin{array}{rl} \times\ldots\ldots\ldots\ldots\ldots & x-d = 0 \\ \hline \end{array}$$

$$x^4 \begin{matrix}-a\\-b\\-c\\-d\end{matrix} x^3 \begin{matrix}+ab\\+ac\\+bc\\+ad\\+bd\\+cd\end{matrix} x^2 \begin{matrix}-abc\\-abd\\-acd\\-bcd\end{matrix} x + abcd = 0$$

34. Hinc fluunt ſequentia THEOREMATA quibus illuſtratur æquationum natura.

 I. Facto

I. Factorum nullus per æquationem factam non repreſentatur. Hinc

Si ... $x^2 = px + q$
erit $y^2 = py + q$. poſito radices eſſe x & y.
Ergo $x^2 + y^2 = p^2 + 2q$ quod dic B (25. *n*. 4)
Si .. $x^3 = px^2 + qx + r$
erit ... $y^3 = py^2 + qy + r$
& ... $z^3 = pz^2 + qz + r$

Ergo .. $x^3 + y^3 + z^3 = pB + qp + 3r$. quod dic C; atque ita poſito $p = A$, erit in æquatione quacunque ſumma quadratorum radicum $pA + 2q = B$; ſumma cuborum $pB + qA + 3r = C$, ſumma quadrato-quadratorum $pC + qB + rA + 4s = D$; ſumma quadrato-cuborum $pD + qC + rB + 5A + 5t = E$. & ſic in infinitum obſervata ſerie litterarum. Sic in æquatione $x^4 = x^3 + 19x^2 - 49x + 30$, erit $p = 1$, $q = 19$, $r = -49$, $s = 30$, & inde orientur $A = 1$, $B = 39$, $C = -89$ & $D = 723$. Nam æquationis illius radices ſunt 1, 2, 3 & $-5 = 1$ & $1 + 4 + 9 + 25 = 39$ & $1 + 8 + 27 - 125 = -89$. & $1 + 16 + 81 + 625 = 723$. *Ut opport.* Sic operatio ipſa locum ex difficillimis Auctoris demonſtrat. (1. 15.).

II. Æquationes tot radices totque ſolutiones quot factores habent; ſed non ſatisfacit problemati ſolutio negativa dum

dum affirmativa quæritur, nec viceverſa.

III. Æquatio 2*di* gradus eſt totum (def. 1) ſectum in duas partes quarum datur ſumma & productum; æquatio tertii eſt totum ſectum in tres partes quarum, præter ſummam & productum, datur ſumma factorum ſub ſingulis binis. In æquatione quarti, datur præterea ſumma factorum ſub ſingulis ternis & ſic porro. Verum terminorum alternorum ſummæ p, r, t, &c. dantur mutato ſigno.

IV. Ubi æquationis radices nullæ ſunt impoſſibiles, tot ſunt radices affirmativæ quot ſignorum in continua ſerie mutationes de + in − & − in +. Cæteræ negativæ ſunt.

V. Eodem redit, ſive mutes ſigna terminorum parium in æquatione ſive in factoribus ipſis; ſed in 2*do* caſu affirmativarum radix maxima fit negativarum minima, & viceverſa: Ergo & in primo.

VI. Eodem redit ſive multiplices aut dividas terminos æquationis per terminos progreſſionis geometricæ cujusvis, ſive factores ipſos per terminos ejusdem progreſſionis inchoatæ; ſed in ſecundo caſu ſervatur æqualitas, ergo & in primo. Hinc æquatio quæcumque affecta fractis, in non affectam converti poteſt.

VII.

VII. Idem fit, ſive in æquationis terminis invertatur x ſive in factoribus ipſis; ſed in ſecundo caſu radices omnes mutantur in reciprocas: ergo & in primo. ut ſi invertatur $x-2=0$ & $x-3=0$ in $\frac{1}{x}-2=0$ & $\frac{1}{x}-3=0$, fiet $6x^2-5x+1=0$. cujus radices jam $\frac{1}{2}$ & $\frac{1}{3}$ quæ prius erant 2 & 3.

35. VIII. Eodem redit ſive ſubſtituas 2, 1, 0 − 1 − 2 &c. pro x in æquatione ſive in factoribus ipſis; ſed in ſecundo caſu radices minuuntur binario, poſito 2 pro x; unitate, poſito 1 pro x; nihilo, poſito 0 pro x; ſed augentur unitate, poſito − 1 pro x; binario, poſito − 2 pro x. Ergo fit idem in primo caſu.

Coroll. 1. Si rectangulo addas ſummam laterum & quadratum unitatis, aggregatum erit factus ex radicibus unitate auctis; ſed ſi rectangulo addas quadratum unitatis & demas ſummam laterum, reſiduum erit factus ex lateribus unitate mulctatis. Hinc patet quid fiat ſi rectangulo addas bis ſummam laterum & quadratum binarii &c.

Coroll. 2. Si ſolido addas ſummam laterum, ſummam rectangulorum ſub ſingulis binis & cubum unitatis, aggregatum erit factus ex lateribus unitate auctis.

Coroll.

Coroll. 3. Si ultimo æquationis cujusvis ordinatæ termino addas *p*, *q*, *r*, *s* &c. ſub propriis ſignis, & inſuper unitatem, aggregatum erit factus ex omnibus radicibus unitate minutis. Id ipſum ſit ſubſtituendo + 1 pro *x*, contrarium vero ſubſtituendo − 1. Hoc Theoremate demonſtratur ſequens.

INVENTIO DIVISORUM.

Si quantitas ſimplex eſt divide eam per minimum ejus diviſorem & quotum per minimum ejus diviſorem, donec quotus reſtet indiviſibilis, ita omnes quantitatis diviſores *primos*, habebis. Dein horum diviſorum ſingulos bines, ternos quaternos duc in ſe & habebis etiam omnes diviſores *compoſitos*. (10)

Eſto 60 dividendum

—2
30
—2
15
—3
5

Ergo 1, 2, 2, 3, 5, ſunt diviſores primi ex 60.

36. Si quantitas poſt hæc manet compoſita, ut $xx - xx - 10x + 6 = 0$, pro *x* ſubſtituo ſigillatim in aliqua columna prima ter-

terminos progreſſionis 1, 0, −1, totque orientur facti quos ſcribo in columna 2da, horumque diviſores omnes in columna tertia, quorum (per Theor. 8. n. 35.) nullus poteſt eſſe radix propoſitæ rationalis niſi fuerit terminus medius alicujus progreſſionis Arithmeticæ decreſcentis ut progreſſio 1ª. Quare progreſſiones occurrentes transfero in columnam 4tam, tantoque diviſionem per medium terminum quem alligo cum ſigno ſuo quantitati x. hoc modo

$x = 1$	4	1, 2, 4	+ 4
$x = 0$	6	1, 2, 3, 6	+ 3
$x = -1$	14	1, 2, 7, 14	+ 2

Erit itaque $x + 3 = 0$ diviſor tentandus & ſuccedit prodeunte quoto $x^2 - 4x + 2 = 0$

Eſto nunc inverſa prioris (Th. 1. n. 35.) æquatio $6x^3 - 10x^2 - x + 1 = 0$

1	4	1, 2, 4	+ 4
0	1	1	+ 1
− 1	14	1, 2, 7, 14	− 2

Differentia + 3 terminorum progreſſionis + 4 + 1 − 2 indicat diviſorem tentandum $x + \frac{1}{3} = 0$ ſeu $3x + 1 = 0$, per quem res ſuccedit, prodeunte quoto $2x^2 - 4x + 1 = 0$. (17. def. 9.)

Rur-

Rurſus eſto $24x^5-50x^4+49x^3-140x^2+64x+30$ dividendum, erit operatio ut ſequitur.

2	42	1, 2, 3, 6, 7, 14, 21 . . .	+3. +3. +7.
1	23	1, 23	+1. −1. +1.
0	30	1, 2, 3, 5, 6, 10, 15, 30	−1. −5. −5.
−1	297	1, 3, 9, 11, 27, 33	−3. −9. −11.
−2	2618	. . . .	. . . . . −17.

Niſi compleas ultimam lineam diviſorum, tres hic occurrunt progreſſiones, quarum termini −1, −5, −5 diviſi per differentiam terminorum 2, 4, 6, dant tres diviſores tentandos $x-\frac{1}{2}$, $x-\frac{5}{4}$ & $x-\frac{5}{6}$ & diviſio per ultimum $x-\frac{5}{6}$ ſeu $6x-5$ ſuccedit, prodeunte quoto $4x^4-5x^3+4x^2-20x-6$.

36. Si nullus occurrit hac methodo propoſitæ quantitatis diviſor $mx\pm p$, fortaſſis occurret compoſitus $mxx\pm px\pm q$, quo in caſu valor ille non poterit non eſſe in ſingulis lineis columnæ diviſorum, ut pote quæ diviſores omnes facti reſultantis ex prima ſubſtitutione continent: Ergo ut de valoribus illis ſubducam mx^2 (quo ſolo turbatur progreſſio propter quadratum x^2) formo quartam aliquam columnam ex tot mx^2 quot ſunt lineæ diviſorum tertiæ & de his omnibus tam poſitive quam negative ſumptis ſubduco

mx^2.

$m\,x^2$ & reſidua ſubductionum ſcribo in columna quinta, in cujus ergo lineis ſingulis non poterit non eſſe $\pm px \pm q$ in quo p & q ſunt conſtantes & ſola x variabilis ut progreſſio prima. Hinc progreſſiones ſingulas 5tæ columnæ adſcribo ſextæ in qua terminus medius $\pm px \pm q$ erit $\pm q$, ob $x = 0$, & ſubducto jam noto $\pm q$ de $\pm px \pm q$ proxime ſuperiori reſiduum erit $\pm p$, propter $x = 1$. Q.E.D.

Nota *Arithm. Univ. pag.* 39. formulam quæ præſcribit *addi & ſubduci diviſores quadratis* mx^2, emendatione indigere, ut amplius patebit conferenti exempla duo ſequentia Auctoris.

Eſto dividendum $x^4 - x^3 - 5x^2 + 12x - 6$.

3	39	1, 3, 13, 39.	9	30, 4, -6, -8, -10, -12, -22 -48	4. -6.
2	6	1, 2, 3, 6.	4	2, -1, -2, -3, -5 -6, -7 -10	2. -3.
1	1	1	1	0, -2	0. 0.
0	6	1, 2, 3, 6.	0	6, 3, 2, 1 - 6 - 3 - 2 - 1	-2. +3.
-1	21	1, 3, 7, 21.	1	20, 6, 2, 0 - 22 - 8 - 4 - 2	-4. +6.
-2	26	1, 2, 13, 26	4	22, 9, -2, -3, -5 - 6 - 17 - 30	-6. +9.

In columna ſexta uſurpo ſucceſſive pro q, -2 & $+3$, pro p, $0+2$ & $0-3$, pro m unitatem. Et ſic pro $mx^2 \pm px \pm q$, habeo diviſores duos tentandos $x^2 + 2x - 2$. & $x^2 - 3x + 3$, per quorum utrumque res ſuccedit.

Rurſus eſto divid $3x^5 - 6x^4 + x^3 - 8x^2 - 14x + 14$ primo rem tento uſurpato 1 pro m, ſed non ſuccedit, quare pro m uſur-

uſurpo 3, ut ſit $3x^2$ primus diviſoris terminus

3	170		27		7.−17
2	38	1, 2, 19, 38	12	26, 7, −10 −11 −13 −14 −31 −50	7.−11
1	10	1, 2, 5, 10	3	7, 2, −1 −2 −4 −5 −8 −13	7.− 5
0	14	1, 2, 7, 14	0	14, 7, 2, 1 −14 −7 −2 −1	7.+ 1
−1	10	1, 2, 5, 10	3	7, 2, −1 −2 −4 −5 −8 −13	7.+ 7
−2	190		12		7.+13
−3					

Expeditionis gratia neglexi diviſores extimos & continuavi progreſſiones, quarum poſteriorem rejicio; quia 27−17 dividit quidem 170, ſed 12+13 non dividit 190. ſed quia 27+7 dividit 170 & 12+7 dividit 190, erit juxta priorem $mxx + px + q = 3x^2 + 7$. diviſor tentandus. Et diviſio ſuccedit prodeunte $x^3 - 2xx - 2x + 2$.

Si nullus inveniri poteſt ea methodo diviſor, conclude propoſitam quantitatem non admittere diviſorem duarum dimenſionum. Si tamen dividendum excedit quintum gradum, forte diviſorem habet $mx^3 + pxx + qx + r$, quem ut eadem methodo detegas, ſcribe mx^3 cum uno e diviſoribus ultimi termini propoſitæ, in columna quarta, quo ſubducto ut prius de tertia, reſiduum diviſum per x erit $px + q$ tot vicibus, quot pro x factæ ſunt ſuppoſitiones. Ita de $x^6 + 12x^3 + 4xx + 4x + 3$ diviſorem $x^3 - 2x^2 + 4x + 3$ elicit

Cl. s'Graveſand. *comment. in Arith. univ. pag.* 195.

38. Si quantitas litteralis eſt (*def.* 6.) nec plures quam duas litteras habeat, ut $x^4 - bx^3 - 5bbx^2 + 12b^3x - 6b$, fiat (per 34. Th. VI.) $x^4 - x^3 - 5xx + 12x - 6$, cujus diviſor $xx + 2x - 2$. Hinc $xx + 2bx - 2bb$ erit diviſor litteralis quæſitus.

Si quantitas litteralis eſt & tres vel plures litteras habet „ Quære diviſores
„ omnes terminorum omnium in quibus
„ aliqua litterarum non eſt, item ter-
„ minorum omnium in quibus alia ali-
„ qua litterarum non eſt, pariter &
„ omnium in quibus tertia quartaque &
„ quinta non eſt; ſi tot ſunt litteræ.
„ Et ſic percurre omnes litteras. Et e
„ regione litterarum colloca diviſores
„ reſpective. Dein vide, ſi in ſerie ali-
„ qua diviſorum per omnes litteras per-
„ gente, partes omnes unicam tantum
„ litteram involventes, tot vicibus re-
„ periantur quotſunt litteræ, una demp-
„ ta, in quantitate propoſita: Et partes
„ duas litteras involventes tot vicibus
„ quot ſunt litteræ, duabus demptis, in
„ eadem quantitate (10). Si ita eſt, par-
„ tes iſtæ omnes, ſub ſignis ſuis ſemel
„ ſumptæ erunt diviſor quæſitus."

Eſto quantitas . . ., $12x^3 - 14bxx + 9cx^2$

$9cxx - 12bbx - 6bcx + 8ccx + 8b^3 - 12bbc - 4bcc + 6c^3$.

Terminorum $8b^3 - 12bbc - 4bc^2 + 6c^3$ in quibus non eſt x invenio (per 36) diviſores ſimplices $2b - 3c$ & $4b - 6c$. Terminorum $12x^3 + 9cx^2 + 8c^2x + 6c^3$, in quibus non eſt b, diviſor ſimplex unicus $4x + 3c$; ac terminorum $12x^3 - 14bx^2 - 12bbx + 8b^3$, in quibus non eſt c, diviſores ſimplices $2x - b$ & $4x - 2b$. Hos diviſores diſpono e regione litterarum, ut hic vides.

x	$2b - 3c$. $4b - 6c$.
b	$4x + 3c$.
c	$2x - b$. $4x - 2b$.

Ergo $4x + 3c - 2b$ erit diviſor quæſitus. Nam ſunt tres litteræ & ſingulæ partes diviſorum non niſi unam involvunt litteram. Ergo partes diviſoris debent in ſerie per omnes litteras x, b, c pergente bis reperiri ſingulæ, idque cum ſignis iiſdem (10); quod fit hic, ſi modo ſigna diviſoris $2b - 3c$ mutentur in $-2b + 3c$; quod ſemper licet.

Eſto nunc ... $12x^5 - 10ax^4 - 9bx^4 - 26aax^3 + 12abx^3 + 6b^2x^3 + 24a^3x^2 - 8aabx^2 - 8abbx^2 - 24b^3x^2 - 4a^3bx + 6aabbx - 12ab^3x + 18b^4x + 12a^4b + 32aab^3 - 12b^5$.

x	$b.\quad 2b^2 - 6a^2.\ 6b^2 - 2a^2.$
a	$4xx - 3bx + 2bb.\ 3x^3 - 6b^3.$
b	$x^2.\quad 3x - 4a.\ 4x^2 + 2ax - 6aa.$

rejicio b, $6b^2$, $-\ 2a^2$, x^2, $3x_3$ & $6b_3$, quia non niſi unam eandem litteram involuunt nec niſi ſemel reperiuntur: ſed retineo $2b^2$, $-\ 6aa$, & $4xx$, quia bis reperiuntur, ut & $-\ 3bx$ & $2ax$, quia duas involvunt litteras nec niſi ſemel ut opportet reperiuntur (10). Ergo $4xx + 2ax - 3bx + 2b^2 - 6a^2$ propoſitam dividet, prodeunte quoto $3x^3 - 4ax^2 - 2aab - 6b^3$.

Si propoſita quantitas ſit mixta (*def.* 6.) ſupple deficientes terminorum dimenſiones per dimenſiones litteræ cujusvis asſumptæ & invento diviſore ut prius litteram aſſumptam dele. Puta ſi in exemplo primo deleatur c, diviſor invenietur $4x - 2b + 3$.

39. Aliquando diviſores facilius quam per has regulas inveniri poſſunt. Ex. gr. Eſto $x^4 - 3ax^3 - 8aa\,x^2 + 18a^3\,x + cx^3 - acxx - 8aacx + 6a^3c$. Quæro communem diviſorem terminorum in quibus eſt c quæ unius eſt tantum dimenſionis, & terminorum in quibus non eſt c, ac diviſor ille, nempe $x^2 + 2ax - 2aa$ dividet totam quantitatem.

„ Cœte-

„ Cœterum „ maximus duarum quan-
„ titatum diviſor communis invenitur
„ perpetua ablatione minoris aut multi-
„ plicis ejus de majori & reliqui de ablato ;
„ nam quæſitus erit diviſor qui tandem ni-
„ hil relinquit. " ita de 667 aufer ter 203
& reliquum 58 ter de 203 & reliquum 29
bis de 58, reſtabitque nihil. Ergo 29
eſt maximus diviſor communis numero-
rum 667. & 203.

„ Ubi quantitates involvunt litteras,
„ opportet illas & reſiduum ordinari
„ (*def.* 7.) & qualibet vice concinnari divi-
„ dendo per ſuos omnes diviſores mono-
„ mios vel qui inſtar horum ſingulos
„ terminos dividunt " quia hi factores
non quæruntur (Ax. III.).

$$\text{Eſto } \frac{x^4 - 3ax^3 - 8aaxx + 18a^3x - 8a^4}{x^3 - ax^2 - 8aax + 6a^3}$$

inventurus maximum communem divi-
ſorum, duc denominatorem in x, ut pri-
mus ejus terminus evadat idem cum pri-
mo termino numeratoris. Dein aufer &
reſtabit $-2ax^3 + 12a^3x - 8a^4$, quod
concinnatum dividendo per $-2a$ evadit
$x^3 - 6a^2x + 4a^3$. Hoc aufer de deno-
minatore, & reſtabit $-ax^2 - 2aax +$
$2a^3$. Quod itidem per $-a$ diviſum fit
$xx + 2ax - 2a^2$. Hoc autem duc in x,

 ut

ut ejus primus terminus evadat idem cum primo noviſſimi ablati $x^3 - 6aax + 4a^3$, de quo aufer & reſtabit $-2axx - 4aax + 4a^3$, quod per $-2a$ diviſum fit etiam $xx + 2ax - 2a^2$; & hoc cum idem ſit ac ſuperius reſiduum, proindeque ablatum relinquat nihil, quæſitus erit diviſor per quem fractio propoſita reduci poteſt ad ſimpliciorem, nempe ad $(xx - 5ax + 4a^2) : (x - 3a)$.

Interdum diviſor prima fronte innoteſcit.

Ex gr. . . . $\dfrac{aadd - ccd^2 - aac^2 + c^4}{4aad - 4acd - 2ac^2 + 2c^3} =$

$$\frac{(a^2-c^2)d^2 + (aa-cc) - c^2}{(a-c)\,4ad + (a-c) - 2cc} = \frac{(a+c)\,(d^2 - c^2)}{4ad - 2cc}$$

Item . . . . . $\dfrac{a^3 - a^2 b - abb + b^3}{aa - bb} =$

$$\frac{aa \times (a-b) - bb \times (a-b)}{(a+b) \times (a-b)} = \frac{a^2 - b^2}{a+b} = a-b. \ (25)$$

Si diviſor communis his pactis non invenitur, certum eſt nullum omnino exiſtere.

REDUCTIO ÆQUATIONUM.

40. *Æquatio finalis* (32) manet indeterminata, ſi plures una quantitates incogni-

cognitas involvat: tot ergo neceſſe eſt dari æquationes medias quot ſunt incognitæ, ut omnes poſſint ad unam denique reduci in qua unica maneat incognita. Sic habitis $2x-5=y$ & $x-2=y$ fit $2x-5=x-2$ & $x=3$. Sic poſitis $a+x-b=y$ & $3b-x=y$, fit $x=(4b-a):2$ in qua non eſt y. Sic habitis $ax=\frac{bbx-b^3}{z}$ & $x=\frac{az}{x-b}$ fit multiplicando $axx=\frac{b^2x-b^3}{x-b}\times\frac{az}{z}$ (29) $=ab^2$ ſeu $x=b$. Univerſaliter...

$$\text{Eſto . } \left.\begin{array}{l} ax+cy=e \\ bx+dy=f \end{array}\right\} \text{ vel } \left\{\begin{array}{l} cy+ax=e \\ dy+bx=f \end{array}\right.$$

$$\text{Erit} \ldots\ y=\frac{af-be}{ad-bc} \ \&\ \ldots\ x=\frac{cf-de}{bc-ad}$$

$$\text{Nam . } \left.\begin{array}{l} abx+bcy=be \\ abx+ady=af \end{array}\right\} \& \left\{\begin{array}{l} cdy+adx=de \\ cdy+bcx=cf. \end{array}\right.$$

$$\text{Ergo . } \frac{af-be}{ad-bc}=y \quad \& \quad \frac{cf-de}{bc-ad}=x. \text{ Q. E. D.}$$

41. Simili modo „ per tres multi-
„ plicationes in cruce atque factorum
„ alterna ſigna mutando, tres invenien-
„ tur incognitæ tribus æquationibus in-
„ volutæ" ut ſequitur.

$$\begin{array}{l} \text{Eſto} \ldots\ ax+dy+gz=l \\ \& \ldots\ bx+ey+hz=m \\ \& \ldots\ cx+fy+kz=n \end{array}$$

 erit

erit.. $z = \frac{aen - afm + bfl - bdn + cdm - cel}{aek - afh + bfg - bdk + cdh - ceg}$

ſeu... $z = \frac{(ae - bd)n + (cd - af)m + (bf - ce)l}{(ae - bd)k + (cd - af)h + (bf - ce)g}$

Nam.. $bx + ey = m - bz)$ $(bx + ey = m - bz$
& .... $ax + dy = l - gz)$ & $(cx + fy = n - kz$

Ergo. $y = \frac{bl - bgz \cdot am + abz}{bd - ae}$ & $y = \frac{bn - bkz \cdot cm + chz}{bf - ce}$

Sed ... $(bl - bgz - am + abz)(bf - ce) = (bn - bkz - cm + chz)(bd - ae)$

Ergo... $z =$ &c. ut ſupra, quo invento dantur x & y per (40) Q. E. D.

Nota $n > m > l$, nec ulla methodo incognitas determinari poſſe ſi a, b, c, d &c ſint quantitates geometrice proportionales.

42. Cæterum cum quantitas exterminanda multarum dimenſionum exiſtit, ad eam ex æquationibus tollendam calculus valde laborioſus nonnunquam requiritur, ſed ejusmodi caſus arguunt ſæpe calculum non ſatis artificioſum eſſe. Sic ex æquationibus $x + y + \frac{y}{x} = 20$ & $xx + yy + \frac{y^4}{xx} = 140$, operoſe nimis quæritur x tolli, *Arithm. Univ. pag.* 61, 62, 63, 78. ut patet ex noſtra reſolutione (46 Prob. XIII).

Por-

Porro deleturus y ex $y^2 = \frac{2xxy}{a} + xx$ & $y^2 = 2xy + \frac{x^4}{aa}$, quis non videt $x = a$? nam $\frac{2aay}{a} + a^2 = 2ay + \frac{a^4}{aa}$. adeoque $y = a$ x. $(1 \pm \sqrt{2})$.

Si tollenda ſit x ex $3xx - 2yx = -4$
& . . . . $x^2 + 5x = 3y^2$

erit xdo in cruce . . . $x = \frac{9yy + 4}{15 + 2y}$, quo valore ſubſtituto pro x in altera ex propoſitis, puta in ſecunda, fiet $69y^4 - 90y^3 + 72yy + 40y + 316 = 0$.

Exterminaturus y de $y^3 - xy^2 - 3x = 0$ & $yy + xy - x^2 + 3 = 0$, duc ſecundam in y (39) & ſubduc; reſtabit $2xy^2 - x^2y + 3y + 3x = 0$. Ergo $y^2 = (x^2y - 3x - 3y) : 2x$. Hinc propoſita ſecunda fit $\frac{x^2y - 3x - 3y}{2x} + xy - xx + 3 = 0$ & $3x^2y - 3y = 2x^3 - 3x$, adeoque $y = (2x^3 - 3x) : (3x^2 - 3)$; quo valore ſubſtituto pro y in eadem, fit $x^6 + 18x^4 - 45xx + 27 = 0$.

Huc referre licet modum tollendi quantitates quotcumque ſurdas ex æquationibus, fingendo eas litteris quibuslibet æquales, ut ſi ſit $\sqrt{ay} - \sqrt{aa - ay} = 2a$

$+\sqrt[3]{ay^2}$, ſcribendo t pro $\sqrt{ay}$, v pro $\sqrt[3]{aa-ay}$ & x pro $\sqrt[3]{ay^2}$ habebuntur æquationes $t-v=2a+x$, $a=ay$, $uu=tt-ay$ & $x^3=ayy$, ex quibus tollendo gradatim t, v & x, reſultabit tandem æquatio libera ab omni Aſymmetria.

Quomodo Problemata ad æquationes redigantur.

43. Cum Artes exemplis facilius quam præceptis addiſcantur Newtono placuit ſequentium Problematum ſolutiones adjungere. jam reſolvimus I, II & III, (13. 15.). Hinc.

PROB. IV.

Viro cuidam inter mendicantes diſtribuere volenti deſunt 8 denarii, quominus det ſingulis denarios 3; dat itaque ſingulis denarios 2 & 3 ſuperſunt. Quæritur numerus mendicantium x?

Numerus denariorum foret $3x$ ſi nihil deeſſet & $2x$ ſi nihil ſupereſſet.

Sed... $3x-8=2x+3$

Ergo... $x-8=3$ & $x=11$. Q.E.I.

PROB.

PROB. V.

Si Tabellarii duo A *&* B, *milliaribus* 59 *diſtantes, tempore matutino obviam eant, quorum* A *conficit milliaria* 7 *in horis* 2 *&* B *milliaria* 8 *in horis* 3; *ſed* B *hora* 1 *ſerius iter inſtituit quam* A: *Quæritur Longitudo itineris quod* A *conficiet priusquam conveniat* B?

Eſto Longitudo illa x & erit $59-x$ Longitudo itineris B.

Pro A . . . $7 : 2 = x : \frac{2x}{7}$

Pro B . . . $8 : 3 = 59-x : \frac{177-3x}{8}$

Sed (*Hypoth.*) $\frac{2x}{7} - 1 = \frac{177-3x}{8}$. Ergo (Ax. III) $16x-56 = 1239-21x$ & $37x = 1295$ & $x = 35$. Q. E. I.

Alter caſus eſt in quo mobilia ambo ad eaſdem plagas tendunt.

EXEMP. *Si quotidie Sol gradum* 1 *conficit & Luna* 13, *& ad tempus aliquod Sol ſit in principio Cancri atque poſt* 3 *dies Luna in principio Arietis: Quæritur locus conjunctionis proxime futuræ?* Reſp. in $10\frac{3}{4}$ gr. Cancri.

Nam cum gradus primus Arietis diſtet

ter

ter 90 gradibus a primo gradu Cancri, ſi Sol percurrat gradus x, Luna metietur gradus $90+x$.

$$\text{Sed} \ldots\ 13 : 1 = 90+x : \frac{90+x}{13}$$

$$\text{Et} \ldots\ 1 : 1 = x : x$$

Ergo $\frac{90+x}{13} = x-3$. (*Hypoth.*) &

$12x = 129$ & $x = 10\frac{9}{12}$ Q. E. D.

PROBL. VI.

Data agentis alicujus poteſtate, invenire quot ejusmodi agentes datum effectum a, in tempore b, producent?

Hoc reſolvit Auctor duabus Analogiis more ſolito: ſed nunquam occurrit regula proportionis duplex, ut nec regula inverſa niſi ex omiſſione unius ex ideis datis. Quod ut manifeſtetur, „ liceat „ in omni Analogia ſcribere ſummam „ extremorum ſub una columna & ſum„ mam mediorum ſub altera.“ Confer. Axioma III. poſtul. V.

EXEMPLUM AUCTORIS. „ Si ſcriba (1, „ quod omittitur) in diebus 8 folia 15 „ deſcribere poteſt, quot ejusmodi ſcri„ bæ (x) requiruntur ad deſcribendum

„ folia

„ folia 405 in diebus 9?" *Reſp.* 24.

Nam ſi *ſcriba* 1) —— 15. 1 *fol.*
dieb. 8)
(x *ſcribæ*
fol. 3 . 45 . 405 —— (9. 1 *Dieb.*

Ergo factus 3+ 8 = 24 = x Q.E D.

Exemplum regulæ inverſæ. „ Si togæ „ ſit longitudo 7 ulnarum & ulnæ lati- „ tudo $\frac{9}{4}$: dic longitudinem x duplicatu- „ ræ cujus latitudo $\frac{7}{8}$.?" *Reſp.* x = 18. *uln.*

Nam ſi *Toga* 1 —— (7 *longit.*
($\frac{9}{4}$ *latit.*

longit. x)
latit. $\frac{7}{8}$) —— 1 *Duplic.*

Axio. III. 4 8. 2

Ergo factus x = 18. Q. E. D.

PROB. VII.

Datis plurium agentium viribus, tempus x determinare in quo datum effectum conjunctim producent?

EXEMPLUM. „ Tres mercenarii opus 1 „ certis temporibus perficiunt, ſcilicet A „ ſemel in ſeptimanis 3, B ter in ſepti- „ manis 8 & C quinquies in ſeptimanis

„ 12:

„ 12 : Qtr. x tempus quo ſimul abſol-
„ vent?"

Dic *Vis* . . $1 \times 3 : 1 = 1 \times x : \frac{1}{3}x$

$1 \times 8 : 3 = 1 \times x : \frac{3}{8}x$

$1 \times 12 : 5 = 1 \times x : \frac{5}{12}x$

ſed opus $1 = \frac{1}{3}x + \frac{3}{8}x + \frac{5}{12}x$. Ergo $x = 1 : (\frac{1}{3} + \frac{3}{8} + \frac{5}{12}) = \frac{8}{9}$ *ſept.* $=$ 6 *dies* $5\frac{1}{3}$ *hor.* Q. E. I.

PROB. VIII.

Diſſimiles duarum pluriumvè rerum miſturas componere, ut res illæ commiſtæ datam inter ſe rationem acquirant ?

Finge x, y, z numeros eſſe per quos, ſi tres datæ miſturæ reſpective multiplicentur, earum ſumma evadat illa quæ deſideratur, eritque hujus & ſequentis problematis reſolutio problema generale (41) ſtantibus pro datis & pro deſideratis litteris iisdem.

Ergo . . $z = \dfrac{(ae-bd)n + (cd-af)m + (bf-ce)l}{(ae-bd)k + (cd-af)h + (bf-ce)g}$

EXEMPLUM. „ Si tres ſint metallorum „ colliquefactorum miſturæ , quarum „ continet pondo primæ argenti unc. „ 12 , æris unc. 1 , & ſtanni unc. 3 ; ſe- „ cundæ pondo argenti unc. 1 , æris unc. „ 12 , & ſtanni unc. 3 ; tertiæ argenti

„ po-

„ pondo unc. o, æris unc. 14, & ſtan-
„ ni unc. 2. Sintque hæ miſturæ ita
„ componendæ, ut pondo compoſitio-
„ nis contineat argenti unc. 4, æris unc.
„ 9, & ſtanni unc. 3."

Pro litteris $a, b, c; d, e, f; g, h, k; l, m, n.$
pone . . . 12, 1, 3; 1, 12, 3; 0, 14, 2; 4, 9, 3.

eritque . . $z = \frac{143 \times 3 - 33 \times 9 - 33 \times 4}{143 \times 2 - 33 \times 14 - 33 \times 0} = \frac{429 - 429}{-176} = 0.$

Sed jam . . . . $ax + dy = l$
Et . . . . . $bx + ey = m$ (40)

Ergo . . . . $y = \frac{am - bl}{ae - bd} = \frac{108 - 4}{144 - 1} = \frac{8}{11}$

& $x = \frac{3}{11}.$

Quare ſi miſceantur $\frac{8}{11}$ partes pondo miſturæ ſecundæ, $\frac{3}{11}$ pondo miſturæ primæ & nihil tertiæ, aggregatum erit pondo continens uncias 4 argenti, 9 æris & 3 ſtanni. Q. E. I.

PROB. IX.

Datis plurium ex iisdem rebus miſturarum pretiis & proportionibus miſtorum inter ſe, pretium cujusvis e miſtis determinare?

EXEMPLUM. „ Emit quidam modios 40
„ tri-

,, tritici, modios 24 hordei, & modios
,, 20 avenæ, ſimul 15 libris, 12 ſolidis:
,, Deinde conſimilis grani emit modios
,, 26 tritici, modios 30 hordei & mo-
,, dios 50 avenæ, ſimul 16 libris: Ac
,, tertio conſimilis etiam grani emit 24
,, modios tritici, 120 modios hordei &
,, 100 modios avenæ, ſimul 34 libris.
,, Quæritur quanti ſit æſtimandus mo-
,, dius cujuscumque grani?" *Reſp.* modius tritici 5 ſolidis, hordei 3 ſolidis & avenæ 2 ſolidis. Nam ſi in formula ſuperiori.

pro $a, b, c; d, e, f; g, h, k; l, m, n$
ſcribo $40, 26, 24; 24, 30, 120; 20, 50, 100; 15\tfrac{3}{5}, 16, 34$

$$\text{fiet } z = \frac{(1200-624)34 + (576-4800)16 + (3120-720)15\tfrac{3}{5}}{(1200-624)100 + (576-4800)50 + (3120-720)20} = \tfrac{2}{20}.$$

adeoque $y = \frac{3}{20}$ & $x = \frac{5}{20} = \frac{1}{4}$ *lb.* Q. E. D.

Problema X.

Datis & miſturæ & miſtorum gravitatibus ſpecificis invenire proportionem miſtorum inter ſe?

Sit e gravitas ſpecifica totius miſturæ $A+B$, cujus partis A gravitas ſpecifica eſt a, partis B gravitas b: ſed gravitas ab-

abſoluta ſeu pondus componitur ex mole corporis & gravitate ſpecifica: hinc erit aA pondus partis A & bB pondus partis B. Adeoque $aA + bB = (A + B) \times e$; indeque $aA - eA = eB - bB$ ſeu $e - b : a - e = A : B$.

EXEMPLUM, „ Sit auri gravitas ut 19, ar„ genti ut $10\frac{1}{3}$ & Coronæ Hieronis ut 17," eritque $17 - 10\frac{1}{3} : 19 - 17$, hoc eſt, $10 : 3 = A : B$, ut moles auri in Corona ad molem argenti, & $10 \times 19 : 3 \times 10\frac{1}{3} = (e-b)\,a : (a-e)\,b$ pondus auri in corona ad pondus argenti, & $190 + 31 : 31 =$ pondus coronæ ad pondus argenti.

PROBLEMA XI.

Si boves a, in tempore b, depaſcant pratum c; & boves d, in tempore e, depaſcant pratum æque bonum f & gramen uniformiter creſcat: Quæritur quot boves x, in tempore g, depaſcent pratum ſimile h?

Si boves a in b depaſcunt c, boves $\frac{abf}{ce}$ in e depaſcent f. & ſi boves a in b depaſcunt c, boves $\frac{abf}{cg}$ in g depaſcent f; puta ſi gramen poſt tempus b non creſceret. Sed poſt tempus b gramen creſcit in pratis f & b, atque per hypotheſim bo-

ves d in e depaſcunt f, hoc eſt, pratum cum incremento. Ergo boves $d - \frac{abf}{ce}$ ſunt qui depaſcunt in e incrementum 1, temporis $e-b$, prati f.

Sed ſi boves $\frac{cde - abf}{ce}$ in e depaſcunt $e-b$, boves $\frac{cde - abf}{cg}$ in g depaſcent $e-b$. & ſi boves $\frac{cde - abf}{cg}$ in g depaſcunt $e-b$, boves $\frac{cde - abf}{cg} \times \frac{g-b}{e-b} = \frac{cdeg - abgf - bcde + ab^2f}{cge - cgb}$ in g depaſcent incrementum $g-b$ ejuſdem prati f. Hos boves adde bobus $\frac{abf}{cg}$, qui depaſcunt f in g dum gramen non creſcere ſupponitur, & prodibit $\frac{cdeg - abgf - bcde + ab^2f}{cge - cgb} + \frac{abf}{cg} \times \frac{e-b}{e-b} = \frac{cdeg - abfg - bcde + abef}{cge - cgb}$, numerus boum quibus paſcendis ſufficit pratum f, per tempus g.

Denique ſi boves $\frac{cdeg - abfg - bcde + abef}{cge - cgb}$ in g depaſcunt f, boves x in g depaſcent h, erit-

eritque $x = \frac{cdegh - abfgh - bcdeh + abefh}{cgef - cgbf}$

Q. E. I.

EXEMPLUM „ Si 12 boves, in 4 ſepti-
„ manis, depaſcant $3\frac{1}{3}$ jugera prati & 21
„ boves, in 9 ſeptimanis, depaſcant 10
„ jugera conſimilis prati; Quæritur quot
„ boves in 18 ſeptimanis depaſcent 24
„ jugera?" *Reſp.* 36.

Nam pro *a*, *b*, *c*, *d*, *e*, *f*; *g*, *h*.
ſcribo . . 12, 4, $3\frac{1}{3}$; 21, 9, 10; 18, 24.

fitque . . $x = \frac{11340 - 8640 - 2520 + 4320}{300}$

$\times \frac{24}{10} = 36$. Q. E. I.

PROBLEMA XII.

Datis ſphæricorum corporum in eadem recta motorum, ſibique occurrentium magnitudinibus & motibus, determinare motus eorundem poſt reflexionem?

Nota corpus utrumque tantum reactione pati quantum agit in alterum & eadem celeritate poſt reflexionem recedere ab invicem qua ante accedebat. His poſitis, ſint corporum *A* & B celeritates *a* & *b* reſpective; & motus (nam idea motus continet ideas celeritatis & molis

 in

in ſe ductas) erunt aA & bB. Et ſi corpora ad easdem plagas tendant & A celerius movens inſequatur B, eſto x decrementum motus aA & incrementum motus bB percuſſione exortum; & poſt reflexionem erunt motus $aA - x$ & $bB + x$ & celeritates $\frac{aA - x}{A}$ ac $\frac{bB + x}{B}$, quarum differentia æquatur $a - b$, differentiæ celeritatum ante reflexionem. Ergo $a - b =$

$$b + \frac{x}{B} - a + \frac{x}{A} \quad \& \quad x = (a - b) \times \frac{2AB}{A + B}.$$

Q. E. I.

EXEMPLUM. „ Si corpora homogenea, A librarum 3, cum celeritatis gradibus 8, & B librarum 9, cum celeritatis gradibus 2, ad easdem plagas tendant. " . . . . .

noto	*Maſſas* . . .	*Celeritates*
pro A	3. *lb.*	8. *gr.*
pro B	9. *lb.*	2. *gr.*

tunc pro A, a, B & b ſcribo 3, 8, 9 & 2 fitque $x = (a - b) \times \frac{2AB}{A + B} = 27$. Ergo $\frac{aA - x}{A} = \frac{24 - 27}{3} = -1$ celereritas negativa qua corpus A recedet, & $\frac{bB + x}{B}$ =

$= \frac{18+27}{9} = 5$, celeritas poſitiva graduum 5, qua B progredietur poſt reflexionem.

In caſu quo A & B obviam irent, ubique ſigno ipſius *b* mutato, fieret -7 celeritas qua poſt reflexionem recederet A & +3 celeritas qua progrederetur B. Nam $\frac{aA-x}{A} = \frac{24-45}{3} = -7$ & $\frac{bB+x}{B} = \frac{-18+45}{9} = 3$.

Nunc ſupplendæ veniunt Progreſſiones, (t).

PROGRESSIO ARITHMETICA.

44. Progreſſio Arithmetica ,, eſt ſeries ,, quantitatum æqui-differentium, ſeu inter quas *differentia communis* eſt". Hujus Progreſſionis index ÷. Eſto *p* primus terminus, *d* differentia communis, *n* numerus terminorum, *s* ſumma omnium & *v* ultimus, & erit ÷ *p*, *p*+*d*, *p*+2*d*, *p*+3*d* &c.

PROPRIETAS I^a^. ,, In omni ÷ ne, terminus ultimus eſt aggregatum ex primo & *differentia communi*, ducta in numerum terminorum unitate mulctatum"

„ tum" nam (*ſupra*) $v = p + 3d = p + d \times (n-1)$. Q. E. D.

PROPRIETAS 2ª. „ In omni ÷ ne, ſum-
„ ma extremorum æquatur ſummæ duo-
„ rum quorumque terminorum æquidi-
„ ſtantium ab extremis."

Demonſtratio. Eſto x ultimus terminus.. Erit $\div p, p+d, p+2d, \ldots x-2d, x-d, x$. & .. $p+x = p+d+x-d = p+2d+x-2d$. &c. Q. E. D. Hinc . . .

THEOR. „ ſumma totius ÷nis eſt pro-
„ ductum ex ſumma extremorum in ſe-
„ mi-numerum terminorum", ſeu $s = (p+v) \times \frac{1}{2} n$.

Harum quinque litterarum s, v, p, d & n, datis tribus, facile eſt reliquas duas ex proprietate 1ª & 2ª eruere. Ita combinando litteras quinque ſub binis & ternis (10) formantur & reſolvuntur problemata ſequentia bis decem.

Data. *Quæſita.* *Solutiones.*

1um, p, d & n .. s & v .. $v = p + d \times (n-1)$

2um, v, d & n .. s & p .. $p = v - d \times (n-1)$

3um, v, p & n .. s & d .. $d = (v-p) : (n-1)$

4um, v, p & d .. s & n .. $n = v - p : d + 1$.

5um, s, d & n .. v & p .. $p = \frac{s}{n} - \frac{1}{2} d \times (n-1)$

6um, s, p & n .. v & d .. $v = 2s : n, -p$

7um, s, p & d .. v & n .. $v = -\frac{1}{2}d + \sqrt{\frac{1}{4}d^2 + 2ds + p^2 - pd}$

8um,

8um, s,v & $n..p$ & $d..p = \frac{2s}{n} - v$.

9um, s,v & $d..p$ & $n..n^2 = (2vn + dn - 2ds) : d$

10um, s,v & $p..d$ & $n..n = 2s : (p+v)$.

Exemplum. „ Dedi Mendico, diebus 10 „ (n), prima die denarios 2 (p). Diffe- „ rentia communis donorum 2 denarii „ (d): Quære ſummam donorum (s) „ & ultimum (v)?" Invento v per *propr.* 1*am* & $s = (p+v) \times n$, per *propr.* 2*am*, facile erit Exemplum idem cæteris problematibus adaptare.

Progressio Geometrica.

45. „ Eſt ſeries quantitatum æqui-contentarum, *ſeu* inter quas *ratio communis* eſt. Hujus index ∺. Sit *ratio communis* r & ſtantibus iiſdem litteris ac in ÷ *ca*, excepta d. Erit ∺ p, pr, pr^2, ... pr^{n-3}, pr^{n-2}, pr^{n-1}

Proprietas 1ª. „ In omni ∺ne productum extremorum æquatur producto „ duorum quorumcumque terminorum „ æquidiſtantium ab extremis". Nam $p \times pr^{n-1} = pr \times pr^{n-2} = pr^2 \times pr^{n-3}$ Q. E. D.

Proprietas 2ª. „ In omni ∺ne terminus ultimus eſt pr^{n-1}". Nam *ſupra* erat ſecundus pr^1 tertius pr^2 &c. Ergo $v = pr^{n-1}$.

Proprietas 3ª. „ In omni $\div\!\div$ ne $s =$ „ $(pr^n - p) : (r-1)$".

Nam eſto $\div\!\div\ p + pr + pr^2 \ldots + pr^{n-1} = s$

erit . . . . . $pr + pr^2 + pr^3 \ldots + pr^n \quad = sr$

Ergo ſubd. . . $pr^n - p = sr - s$ & $s = (pr^n - p) : (r-1)$ Q. E. D.

Ex proprietate 2ª & 3ª rurſus formantur & reſolvuntur Problemata ſequentia bis decem.

Datæ Quæſitæ. Solutiones.

1um, p, r & n . . s & v . . $v = pr^{n-1}$ & $s = (pr^n - p) \times (r-1)$

2um, v, r & n . . s & p . . $p = vr : r^n$.

3um, v, p & n . . s & r . . $r = \sqrt[n-1]{v : p}$.

4um, v, p & r . . s & n . . $s = (vr - p) : (r-1)$.

5um, s, r & n . . v & p . . $p = (rs - s) : (r^n - 1)$

6um, s, p & n . . v & r . . $r = \sqrt[n]{(rs + p - s) : p}$

7um, s, p & r . . v & n . . $v = (rs + p - s) : r$

8um, s, v & n . . p & r . . $\frac{r^{n-1}}{r^n - 1} = v : s$

9um, s, u & r . . p & n . . $p = vr - sr + s$

10um, s, u & p . . r & n . . $r = (s-p) : (s-v)$.

Coroll. In omni $\div\!\div$ ne quadratum termini ex mediis divíſum per præcedentem æquatur ſequenti & diviſum per ſequentem æquatur præcedenti. Nam eſto $\div\!\div$ p, pr, pr^2, erit $p^2r^2 : p = pr^2$ & $p^2r^2 : pr^2 = p$.

46. Nunc in ∺ ne quacumque eſto n numerus terminorum, p ſumma omnium, B ſumma quadratorum, q ſumma factorum ſub ſingulis binis & s ſumma terminorum parium, eritque THEOREMA „ $qs-$ „ $q=0$, dum n eſt numerus impar". Nam eodem redit ſive ducas s in p, ſive ducas terminos pares in terminos ſingulos ∺ nis: ſed in 2do caſu reſultat q (25 Th. 4). Ergo & in primo $ps=q$ & $ps-q=0$. Q.E.D. Verum ubi n eſt numerus par „ erit $ps-q$ „ ſumma quadratorum ultimorum ∺ nis „ terminorum, ſcilicet duorum ſi $n=4$, „ trium ſi $n=6$, quatuor ſi $n=8$, & ſic „ porro". Sed ſi $n=4$ & s exprimat terminos medios „ erit $ps-q$ ſumma qua- „ dratorum mediorum". Quæ demonſtrantur *ut ſupra.* His poſitis ad Auctorem redimus.

PROB. XIII.

Invenire tres numeros continue proport., quarum ſumma 20 & ſumma quadratorum 140?

Eſto ∺ $x+y+z=20$, erit (25. Th. 4.) $q=(400-140):2=130$. & (46) $20y-130=0$. Ergo $y=13:2$. Sed $xz=yy=169:4$ & $x+z=20-13:2=27:2$.

 Ergo

Ergo (15) x vel $z = (27 \pm \sqrt{726-676}) : 4$
Q. E. I.

PROB. XIV.

Invenire quatuor quantitates continue proport, quarum datur ſumma p & ſumma mediarum a?

Eſto $\div \frac{xx}{y} + x + y + \frac{yy}{x} = p$, erit $pxy = x^3 + x^2y + y^2x + y^3 = (x^2 + y^2) \times (x + y)$. Ergo $pxy : a = a^2 - 2xy$. (25. Th. 4). & $pxy + 2axy = a^3$. & $xy = a^3 : (p + 2a)$. & (19) x vel $y = (a \pm a\sqrt{(p^2 - 2a) : (p^2 + 2a)}) : 2$. Q. E. I.

EXEMPL. Eſto $a = 12$ & $p = 32$, erit x vel $y = 6 \pm 6\sqrt{\tfrac{1}{7}}$ & v vel $z = 10 \pm \sqrt{99\tfrac{1}{7}}$.

PROB. XV.

Invenire quatuor quantitates continue proport, quarum datur ſumma p & ſumma quadratorum B?

Eſto $\div v + x + y + z = p$ & $x + y = s$. erit (per præc.) $xy = s^3 : (p + 2s)$. Sed (46) $x^2 + y^2 = ps - q = s^2, -2s^3 : (p + 2s)$. Ergo $p^2s + 2pss - qp - 2qs = ps^2 + 2s^3 - 2s^3$. & $ps^2 \begin{smallmatrix} + p^2 \\ - 2q \end{smallmatrix} s - qp = 0$ & $s = - (B +$

$\sqrt{B^2+4p^2q})$: $2p$. Q. E. I.

EXEMPL. Eſto $p=30$ & $B=340$ eruntque $x+y=12$, $v+z=18$ & $xy=32$.

PROBLEMA XVI.

Si penſio annua librarum a, per 5 annos proxime ſequentes ſolvenda, ematur parata pecunia c: quæritur quanti æſtimanda ſit uſura uſuræ 100 librarum per annum?

Pone $1-x$ uſuram uſuræ pecuniæ x in anno, hoc eſt quod pecunia 1, quam non niſi poſt annum teneor ſolvere, valeat x paratæ pecuniæ. Sed $1:x=a:ax=ax:ax^2=ax^2:ax^3$ &c, Ergo pecunia a poſt annum ſolvenda valebit ax, poſt 2 annos ax^2, poſt 3 ax^3, poſt 4 ax^4, & poſt 5 ax^5. Adde hæc & fiet $ax^5+ax^4+ax^3+ax^2+ax=c$ ſeu $x^5+x^4+x^3+x^2+x=c:a$, æquatio quinque dimenſionum, cujus ope cum x proxime per regulas ſequentes inventum fuerit: pone $x:1=100:y$. Et erit $y-100$ uſura uſuræ 100 librarum per annum.

Quomodo Quæſtiones Geometricæ ad æquationem redigantur?

Quæſtiones geometricæ eadem facili-

tate

tate iiſdemque legibus ad æquationes nonnunquam rediguntur: quippe Logica multiplex non eſt. Ut ſi linea recta a in extrema & media ratione ſecanda ſit, hoc eſt ita ut x^2 quadratum maximæ partis ſit æquale rectangulo ſub tota & minore parte contento, dic $\because a : x = x : a - x$. eritque $x^2 = a^2 - ax$ & (15) $x = (-a + a\sqrt{5}):2$. Q. E. F. [Vel ſi quæris triangula rect, quorum latera ſint rationalia, in infinitum, dic latus minus $a - b$, hipotenuſam $a + b$ & x latus medium, eritque $aa - 2ab + bb + xx = aa + 2ab + bb$ (Elem. l. 1. 47) ergo $xx = 4ab$ & latus med. $x = 2\sqrt{ab}$. Q. E. I. Ex. gr. Eſto $\sqrt{a} = 2$, $\sqrt{b} = 1$, erit $a - b = 3$, $2\sqrt{ab} = 4$ & $a + b = 5$ *ut opport.* Sed eſto $\sqrt{a} = 2$ & $\sqrt{b} = \frac{1}{2}$, erit $a - b = 3:4$, $2\sqrt{ab} = 4:4$ & $a + b = 5:4$. *ut opport.* Porro ſi vis triangulum rect. datis in numeris integris lateribus $\sqrt{a} + \sqrt{b} + \sqrt{c}$ in aliud ejuſdem areæ, cujus latera rationalia, reſolvere: pone differentiam ſegmentorum baſis $x = (a - b) : 2c$ (*cor. prob. II. pag.* 78) eſſe latus minus. Per hoc divide duplam aream $\sqrt{ab}$, erit quotus latus medium y. ut ſi $\sqrt{a} + \sqrt{b} + \sqrt{c} = 3 + 4 + 5$ erit $x = 7:10$. $y = 120:7$ & (Elem. I. 47.) hipotenuſa $z = 1201:7$. Q. E. F.]
Sed in rebus geometricis, idearum media-

diarum inventio plerumque pendet a variis linearum poſitionibus & relationibus complexis & in earum electione cujusque ingenium ſibi debet eſſe, ut in aliis ſcientiis, operandi norma.

Propoſito igitur aliquo Problemate, „ quantitates quas involvit confer &, nullo inter datas & quæſitas habito diſcrimine, perpende quomodo aliæ ex aliis „ dependeant; ut cognoſcas quænam ſi aſſumantur, via breviori (13) dabunt „ cœteras; ad hanc ſemper metam collimando, ut *Schema* problemati accommodatum in triangula vel data, vel „ ſimilia, vel rectangula, quorum proprietates notæ ſunt, reſolvatur."

PROB. I.

Data recta terminata AB *a cujus extremitatibus rectæ* AC, BC *ducuntur in angulis datis* CAB, ABC: *invenire* CD *altitudinem concurſus* C *ſupra datam* AB. TAB. I. FIG. I.

Sit AB $=a$ | CD $=y$ | Ob ang. CAD datum, datur (ex Tab. ſin & tangentium) ratio inter lineas CD & AC, quam pone ut d ad e, eritque $d:e=y:\frac{ey}{d}$ ($=$ AC). Similiter, ob datum ang. CBD, datur

datur ratio inter CD & CB, quam pone ut d ad f. eritque $d : f = y : \frac{fy}{d}$ ($=$ CB).

Sed $(fy + ey) : d = a$. Ergo $y = ad : (e + f)$.

PROB. II.

TAB. I. FIG. I. *Cujuslibet trianguli* ACB, *datis lateribus* AC, BC *&* *baſi* AB *quam perpendiculum* CD *ab angulo verticali ſecet in* D, *invenire ſegmenta* AD *ac* BD.

Sit AC $= a$ | CD$q = ($AC$q -$ AD$q) = a^2 - x^2$.
BC $= b$ | Sed CD$q = ($CB$q -$ BD$q) =$
AB $= c$ | $b^2 - c^2 + 2cx - x^2$ (ob $c - x =$ BD)
AD $= x$ | Ergo $x = (a^2 + c^2 - b^2) : 2c = \frac{1}{2}c$
$\pm (aa - bb) : 2c$. (*Elem.* II. 13.)

THEOR. „ In triangulo quocumque dif-
„ ferentia ſegmentorum baſis eſt diffe-
„ rentia quadratorum laterum diviſa
„ per baſim.

PROB. III.

TAB. I. FIG. 2. *Trianguli rect.* ABC *perimetro* a *&* *area* b *datis, invenire hypotenuſam* BC.

Sit AC $= x$ | $x + y = a - z$ & $xy = 2b$. Er-
AB $= y$ | go (25. Th. 4) $a^2 - 2az + z^2 - 4b$
BC $= z$ | $= zz$ & $z = (a^2 - 4b) : 2a$.

PROB.

PROB. IV.

Datis trianguli rect. perimetro a & perpendiculo b, invenire triangulum. TAB. I. FIG. 1.

Sit CD $= x$ | $x + y = a - z$ & $xy = bz$. Ergo (25. Th. 4.) $a^2 - 2az + zz - 2bz = z^2$ & $z = a^2 : (2a + 2b)$
AC $= y$
AB $= z$

quod notum dic c. Sed $x + y = a - c$ & $xy = bc$. Ergo (15) x vel $y = (a - c \pm \sqrt{aa - 2ac + cc - bc}) : 2$.

PROB. V.

Datis trianguli rect. baſi AB *& ſumma perpendiculi & laterum* CA + CB + CD, *invenire triangulum.*

Eſto AB $= b$ | $x + y = a - z$ & $xy = bz$. Ergo (25. Th. 4.) $a^2 - 2az + z^2 - 2bz = b^2$. & (15) $z = a + b - \sqrt{2ab + 2bb}$.
CB $= x$
CA $= y$
CD $= z$
& $y + z = a$

Geometrice ſic. „ in omni „ triangulo rect. de ſumma perimetri „ & perpendiculi, aufer mediam „ proport. inter eandem ſummam & du- „ plum baſis & reſtabit perpendiculum.

PROB.

PROB. VI.

TAB. I. FIG. I. *Datis in triangulo rect. ABC ſumma laterum $a = AC + BC$ & perpendiculo $b = CD$, invenire triangulum.*

Sit $AC = y$ | $xy = bz$. Et (25. Th. 4.)
$BC = x$ | $a^2 - 2bz = zz$; Ergo $z = -b +$
$AB = z$ | $\sqrt{bb + aa}$, quo invento datur $xy = bz$ & $x + y = a$.

PROB. VII.

TAB. I. FIG. I. *In triangulo rect. datis ſumma laterum a & ſumma b perpendiculi & baſis, invenire triangulum.*

Sit $AC = x$ | $x + y = a$ & $b - z = u$. & $bz -$
$CB = y$ | $z^2 = uz = xy$. Sed (25. Th. 4.)
$AB = z$ | $a^2 - 2bz + 2zz = z^2$; Ergo
$CD = u$ | $z^2 - 2bz + a^2 = 0$ & $z = b \pm$
$\sqrt{bb - aa}$ & $u = \sqrt{bb - aa}$.

PROB. VIII.

TAB. I. FIG. 3. *Trianguli cujusvis ABC datis area, perimetro & uno angulorum A, cætera determinare.*

Eſto

Eſto $\sqrt{a}$ perim. $\sqrt{b}$ area $\sqrt{x} = AC$. Ab ignotorum angulorum alterutro C, demitte perpend. CD, & ob ang. datum A, erit (*vid. Prob.* I.) $AC : CD = \sqrt{d} : \sqrt{e}$; Ergo $CD = \sqrt{ex : d}$ & $2\sqrt{bd : ex} = AB$. Adde $AD = \sqrt{ACq - CDq}$ & erit $BD = 2\sqrt{bd : ex} + \sqrt{x - ex : d}$.

Sed $BDq + CDq = BCq$. Ergo $BCq = 4bd : ex + 4\sqrt{bd : e}, - b + x$.

Verum $BC = \sqrt{a} - AC - AB = \sqrt{a} - \sqrt{x} - 2\sqrt{bd : ex}$. Ergo rurſus $BCq = a + x, + 4bd : ex, - 2\sqrt{ax} - 4\sqrt{abd : ex} + 4\sqrt{bd : e}$. Hinc $a - 2\sqrt{ax} - 4\sqrt{abd : ex} + 4\sqrt{bd : ex} - 4\sqrt{bd : e}, - b = 0$. Pro cognitis $a + 4\sqrt{bd : e} - 4\sqrt{bd : e}, - b$, ſcribo $4\sqrt{af}$, fitque $-2x\sqrt{a} - 4\sqrt{abd : e} + 4\sqrt{afx} = 0$. ſeu $x = 2\sqrt{fx} - 2\sqrt{bd : e}$. Ergo (15) $\sqrt{x} = \sqrt{f} \pm \sqrt{f - 2\sqrt{bd : e}}$. Q. E. I. & exprimit tam crus AB quam crus BC. quia illa duo ſimiliter ſe habent ad omnes conditiones problematis; atque horum ſumma $2\sqrt{f}$ ſubducta de perimetro relinquit tertium latus $BC = \sqrt{a} - 2\sqrt{f}$.

PROB. IX.

Datis altitudine, baſi & ſumma laterum invenire triangulum. TAB. II. FIG. [illegible]

Sit altit. CD $= \sqrt{a}$ | Erit latus maj. BC
baſis AB $= 2\sqrt{b}$ | $= \sqrt{c} + \sqrt{z}$ & latus
ſummæ lat: $= 2\sqrt{c}$ | minus AC $= \sqrt{c} - \sqrt{z}$.
diff. lat: $= 2\sqrt{z}$ | Ergo $\sqrt{BCq - CDq}$

$= \sqrt{c + 2\sqrt{cz} + z - a} =$ BD. Et $\sqrt{ACq - CDq} = \sqrt{c - 2\sqrt{cz} + z - a} =$ AD. Sed BD − AB $= \sqrt{c + 2\sqrt{cz} + z - a} - 2\sqrt{b} =$ AD. Ergo (*Ax.* II. III.) $c - 2\sqrt{cz} + z - a = c + 2\sqrt{cz} + z - a - 4\sqrt{b} \times \sqrt{c + 2\sqrt{cz} + z - a} + 4b$. & $bc - ba - b^2 = cz - bz$. Et $\sqrt{z} = \sqrt{b} \times \sqrt{1, -a : (c - b.)}$

PROB. X.

TAB. I. FIG. 4. *Datis baſi* AB, *ſumma laterum* AC + BC *& angulo verticali* C, *determinare latera.*

Sit baſis $= a$ | Ab alterutro ignotorum
ſemiſum. lat. $= b$ | ang. A, ad latus oppoſi-
ſemi diff. $= x$ | tum BC, demitte perpend.
erit BC $= b + x$ | AD & ob ang. C datum;
AC $= b - x$ | erit (*vid.* Prob. I.) AC:

CD $= d : e$. Ergo CD $= (b - x)\frac{e}{d}$ ſed (*per* 13. II. *Elem.*) CD $= (ACq - ABq + BCq) : 2BC$. Ergo $x = \sqrt{\frac{da^2 + 2eb^2 - 2db^2}{2d + 2e}}$ unde dantur latera.

Con-

Concluſio foret concinnior, ſi anguli ad baſim quærerentur. Utpote duc EC datum ang. biſecans & baſi occurrens in E, & erit AB : AC + BC (= AE : AC) = ſin. ang. ACE : ſin. ang. AEC. & ab ang. AEC ejusque complemento BEC, ſi ſubducas $\frac{1}{2}$ anguli C, reſtabunt ang. ABC & BAC.

PROB. XI.

Datis Trianguli lateribus invenire angulos? TAB. I. FIG. 5.

Dentur AB = a	Quæratur ang. A. demiſſo ad AB perpend CD
AC = b	
BC = c	quod angulo iſti opponitur, erit (*per cor. prob.* II.) $AD = \frac{1}{2}a + \frac{bb-cc}{2a}$; unde prodit Hocce *Theorema primum.*

I. Ut AB : AC + BC = AC − BC : N & $\frac{1}{2}$ AB + $\frac{1}{2}$ N = AD. Ut AC ad AD ita radius ad coſinum anguli A.

Ad hæc $DCq = ACq - ADq = (2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4) : 4a^2 = (a+b+c) \times (a+b-c) \times (a-b+c) \times (-a+b+c) : 4a^2$. Unde multiplicatis numeratoris & denominatoris radicibus per b, conflatur hocce *Theorema ſecundum.*

II. Ut $2ab$ ad med. prop. inter $(a+b+c) \times (a+b-c)$ & $(a-b+c) \times (-a+b+c)$, ita radius ad ſinum anguli A.

Inſuper in AB cape AE $=$ AC, & age CE & erit ang. ECD $= \frac{1}{2}$ anguli A. Auſer AD de AE & reſtabit DE $= b - \frac{1}{2} a - \frac{bb+cc}{2a} = (c^2 - a^2 + 2ab - b^2) : 2a = (c+a-b) \times (c-a+b) : 2a$. Unde DE$q$ $= (c+a-b) \times (c+a-b) \times (c-a+b) \times (c-a+b) : 4aa$. Et hinc conſit *Theor. tertium, quartumve.*

III. Ut $2ab$ ad $(c+a-b) \times (c-a+b)$ (ita AC ad DE) ita radius ad ſinum verſum anguli A.

IV. Et ut med. prop. inter $a+b+c$ & $a+b-c$ ad med. prop. inter $c+a-b$ & $c-a+b$ (ita CD ad DE) ita radius ad tangentem $\frac{1}{2}$ anguli A, vel dimidii cotangens ad radium.

Præterea eſt CEq $=$ CDq $+$ DEq $= (2ab^2 + bc^2 - ba^2 - b^3) : a = \frac{b}{a} \times (c+a-b) \times (c-a+b)$. Unde *Theor. quintum & ſextum.*

V. Ut med. prop. inter $2a$ & $2b$ ad med. prop. inter $(c+a-b)$ & $c-a+b$, vel ut 1 ad med. prop. inter $(c+a-b) : 2a$ & $(c-a+b) : 2b$ (ita AC ad $\frac{1}{2}$ CE vel CE ad DE) ita radius ad ſinum $\frac{1}{2}$ anguli A.

VI.

VI. Et ut med. prop. inter $2a$ & $2b$ ad med. prop. inter $a+b+c$ & $a+b-c$ (ita CE ad CD) ita radius ad coſinum $\frac{1}{2}$ anguli A.

Si præter angulos deſideretur etiam area trianguli, duc CDq in $\frac{1}{4}ABq$ & radix, ſcilicet $\frac{1}{4}$ $\sqrt{(a+b+c) \times (a+b-c) \times (a-b+c) \times (-a+b+c)}$ erit area illa quæſita.

PROB. XII.

Trianguli cujusvis rectilinei datis lateribus & baſi, invenire ſegmenta baſis, perpendiculum, aream & angulos? TAB. I. FIG. 6.

Trianguli ABC dentur latera AC, BC & baſis AB. Biſeca AB in I & in ea, utrinque producta, cape AF & AE æquales AC, atque BG & BH æquales BC. Junge CE, CF & à C ad baſem demitte perp. CD. Et (per Theor. prob. II.) erit $(ACq - BCq) : 2AB = DI$. Ergo $2AB : AC + BC = AC - BC : DI$, quod eſt Theor. pro determinandis ſegmentis baſis.

De IE, hoc eſt de $AC - \frac{1}{2}AB$, aufer DI, & reſtabit $DE = (BCq - ACq + 2AC) \times (AB - ABq) : 2AB$, hoc eſt $(BC + AC - AB) \times (BC - AC + AB) : 2AB$ ſive $HE \times EG : 2AB$. Aufer DE de FE ſive 2AC & reſtabit $FD = (ACq + 2AC \times AB + ABq - BCq) :$

 2AB,

2AB, hoc eſt $=$ (AC + AB + BC) x (AC + AB − BC): 2AB, ſive FG x FH: 2AB. Et cum ſit CD med. prop. inter DE & DF, CE med. prop. inter DE & EF, ac CF med. prop. inter DF & EF: erit CD $= \sqrt{}$(FG x FH x HE x EG): 2AB, CE $= \sqrt{}$(AC x HE x EG): AB & CF $= \sqrt{}$(AC x FG x FH): AB, Duc CD in $\frac{1}{2}$ AB & habebitur area $= \frac{1}{4} \sqrt{}$ FG x FH x HE x EG.) Pro angulo vero A determinando prodeunt Theoremata multiplicia, ſc.

I. 2AB x AC: HE x EG ($=$ AC: DE) $=$ rad. ad ſin. verſum anguli A.

II. 2AB x AC: FG x FH ($=$ AC: FD) $=$ rad ad coſin. verſ. anguli A.

III. 2AB x AC: $\sqrt{}$(FG x FH x HE x EG) ($=$ AC: CD) $=$ rad ad ſin. A.

IV. $\sqrt{}$(FGxFH): $\sqrt{}$(HExEG) ($=$ CF:CE) $=$ rad. ad tang. $\frac{1}{2}$ A.

V. $\sqrt{}$(HE x EG): $\sqrt{}$(FGxFH)($=$ CE : FC) $=$ rad. ad cotang. $\frac{1}{2}$ A.

VI. 2$\sqrt{}$(ABxAC): $\sqrt{}$(HExEG)($=$ FE:EC) $=$ rad. ad ſin. $\frac{1}{2}$ A.

VII. 2$\sqrt{}$(ABxAC): $\sqrt{}$(FGxFH)($=$ FE:FC) $=$ rad. ad coſin. $\frac{1}{2}$ A.

PROB. XIII

TAB. I. FIG. 7. *Datum angulum* CBD *recta data* CD *ſubten-*

tendere; ita ut ſi a termino iſtius rectæ D ad punctum A in recta CB producta datum agatur AD, fuerit angulus ADC æqualis angulo ABD?

Sit $CD = \surd a$, $AB = \surd b$, $BD = \surd x$. $BD : BA = CD : DA = \surd \frac{ab}{x}$. Demitte perpend DE, erit $BE = (BDq - ADq + BAq) : 2BA = (x - ab : x, + b) : 2\surd b$. Ob datum Ang. DBA pone $BD : BE = \surd b : \surd e$ & habebitur iterum $BE = \surd \frac{ax}{b}$. Ergo $(x - \frac{ab}{x} + b) : 2\surd b = \surd \frac{ex}{b}$. Et $x^2 - 2x\surd ex + bx - ab = 0$, æquatio 4ti gradus, ut patet ſcripto x pro $\surd x$.

PROB. XIV.

Invenire triangulum ABC cujus tria latera AB, AC, BC & perpendiculum DC ſunt in Arithmetica progreſſione? TAB. I. FIG. 1.

Eſto $CD = x$, $CB = x + d$, $AC = x + 2d$, $AB = x + 3d$.

$AD \; (= \surd \overline{ACq - DCq}) = \surd \overline{4dx + 4d^2}$

$BD \; (= \surd \overline{BCq - DCq}) = \surd \overline{2dx + d^2}$

 Ergo

Ergo $x + 3d = \sqrt{4dx + 4d^2} + \sqrt{2dx + d^2}$, & quadratis &c partibus fit $x^2 + 4d^2 = \sqrt{(4dx + 4d^2) \times (8dx + 4d^2}$. Et his iterum quadratis $x^3 - 24d^2x - 48d^3 = 0$, quæ dividendo per $2d$ (34. Th. VI.) fit $x^3 - 6x - 6 = 0$.

PROB. XV.

TAB. I. FIG. I. *Invenire triangulum* ABC *cujus tria latera* AB, AC, BC & *perp.* CD *ſunt in Geometrica progreſſione?*

Dic CD $= 1 : \sqrt{x}$ BC $= 1$. AC $= \sqrt{x}$. erit AB $= x$.	Eſt AD $= \sqrt{x - 1 : x}$ & BD $= \sqrt{1 - 1 : x}$. Ergo AB $= x = \sqrt{x - 1 : x} + \sqrt{1 - 1 : x}$ Et $x -$

$\sqrt{1 - 1 : x} = \sqrt{x - 1 : x}$. Et $x^2 - 2x\sqrt{1 - 1 : x} + 1 - 1 : x = x - 1 : x$, adeoque $x^2 - x + 1 = 2x\sqrt{1 - \frac{1}{x}}$. Ergo $x^4 + x^2 + 1 - 2x^3 + 2x^2 - 2x = 4x^2 - 4x$ & $x^4 - 2x^3 - x^2 + 2x + 1 = 0$. Sed hujus diviſor (36) $x^2 - x - 1 = 0$. Ergo (15) $\sqrt{x} = \sqrt{(1 + \sqrt{5}) : 2}$.

PROB. XVI.

TAB. I. FIG. I. *Super data baſi* AB *triangulum* ABC *conſti-*

ſtituere, cujus vertex C erit ad rectam EC poſitione datam, baſis autem medium exiſtet Arithmeticum inter latera?

Dic AB $= \sqrt{a}$ FE $= \sqrt{b}$ BC-AB $= \sqrt{x}$	Baſis AB biſecetur in F & producatur donéc occurrat in E & ad ipſam

demittatur perp. CD, erit BC $= \sqrt{a} + \sqrt{x}$, AC $= \sqrt{a} - \sqrt{x}$, & (per 13. II. Elem.) erit BD $=$ (BCq − ACq + ACq) : 2AB $= 2\sqrt{x} + \frac{1}{2}a$. Ergo FD $= 2\sqrt{x}$, & DE $= \sqrt{b} + 2\sqrt{x}$ & CD ($= \sqrt{CBq - BDq}$) $= \sqrt{\frac{3}{4}a - 3x}$. Sed ob datas poſitiones rectarum CE & AB datur angulus CED, adeoque & ratio DE ad CD, quæ ſi ponatur $\sqrt{d}$ ad $\sqrt{e}$ dabit analogiam $\sqrt{d} : \sqrt{e} = \sqrt{b} + 2\sqrt{x} : \sqrt{\frac{3}{4}a - 3x}$. Ergo $\sqrt{be} + 2\sqrt{ex} = \sqrt{\frac{3}{4}ad - 3dx}$. Et $be + 4ex + 4e\sqrt{bx} = \frac{3}{4}ad - 3dx$. Et $4ex + 3dx = -4e\sqrt{bx} + \frac{3}{4}ad - eb$ Et (per 15. & 34. Th. VI.) $\sqrt{x} = (-2e\sqrt{b} + \sqrt{3ade - 3bde + \frac{9}{4}add}) : (4e + 3d)$. Dato autem $\sqrt{x}$ datur BC $= \sqrt{a} + \sqrt{x}$ & AC $= \sqrt{a} - \sqrt{x}$.

PROB. XVII.

Datis parallelogrammi cujuscumque lateribus AB, BD, DC *&* AC *& una li-* TAB. I. FIG. 8.

nea diagonali BC, *invenire alteram diag.* AD?

Sic E concurſus Diagonalium & ad Diag. BC demitte normalem AF, & (per 13. II. Elem.) erit $(ACq - ABq + BCq) : 2BC = CF$, atque etiam $(ACq - AEq + ECq) : 2EC = CF$. Sed $EC = \frac{1}{2} BC$ & $AE = \frac{1}{2} AD$; Ergo $(ACq - ABq + BCq) : 2BC = (ACq - \frac{1}{4} ADq + \frac{1}{4} BCq) : BC$ & facta reductione $AD = \sqrt{2ACq + 2ABq - BCq}$.

Unde obiter, „ in quolibet parallelogr. ſumma quadratorum laterum æquatur ſummæ quadratorum Diagonalium."

PROB. XVIII.

TAB. I. FIG. 9. *Datis Trapezii ABCD angulis, perimetro & area determinare latera?*

Sit perim. $= a$ area $= bb$ $AB = x$ $BC = y$	Produc latera duo AB & BC donec concurrant in E, & ob ang. omnes datos, dantur

rationes BC ad CE & BE, quas pone d ad e & f, & erit $CE = ey : d$ & $BE = fy : d$; adeoque $AE = x, + fy : d$. Dantur etiam rationes AE ad AD ac DE, quas pone g & h ad d, & erit $AD = (dx + fy) : g$ & $ED = (dx + fy) : h$ adeoque $CD =$

(dx

$(dx+fy):h, -ey:d$. Et $a = x+y+(dx+fy):g+(dx+fy):h, -ey:d$ in qua, ſcripto $p:r$ pro dato $1+d:g+d:h$ & $q:r$ pro dato $1+f:g, +f:h, -e:d$, habebitur æquatio $(px+qy):r = a$.

Ad hæc ob datos ang. omnes datur ratio BCq ad triangulum BCE, quam pone m ad n, eritque $BCE = ny^2:m$. Datur & ratio AEq ad triangulum ADE, quam pone m ad d, eritque $ADE = (d^2x^2+2dfxy+f^2y^2):dm$. Quare $ADE - BCE = bb = (d^2x^2+2dfxy+f^2y^2-dny^2):dm$. Sed æquatio ſuperior dat $x = (ra-qy):p$, quo valore ſcripto pro x in inferiori, provenit $(dr^2a^2-2dqray+dq^2y^2):ppm, +(2afry-2fqy^2):pm+(f^2y^2-dny^2):dm = bb$. Et abbreviatis terminis ſcribendo s pro dato $(dq^2:p^2, -2fq:p, -f^2:d, -n$, & st pro dato $(+adqr):p^2, -afr:p$, ac stv pro dato $bbm, -dr^2a^2:p^2$, oritur $y^2 = 2ty+tv$ ſeu $y = t+\sqrt{tt+tv}$.

PROBLEMA XIX.

Piſcinam ABCD *perambulatorio* ABCDEFGH, *datæ areæ & ejuſdem ubique latitudinis, circumdare?* TAB. I. FIG. 10.

Sit

Sit latit. $= x$ | A punctis A, B, C, D ad area $= aa$ | lineas EF, FG, GH & HE demiſſis perpend AK, BL, BM, CN, CO, DP, DQ, AI perambulatorium dividatur in quatuor trapezia & in quatuor parallelogramma AL, BN, CP, DI. latitudinis x & ejusdem longitudinis cum lateribus dati trapezii. Sit ergo ſumma laterum $AB + BC + CD + DA = b$. & erit bx ſumma parallelogrammorum.

Porro ductis AE, BF, CG, DH; cum ſit $AI = AK$. erit ang $AEI = AEK = \frac{1}{2}$ IEK ſive $\frac{1}{2}$ DAB. Datur ergo ang. AEI. & proinde ejus ratio AI ad IE, quam pone d ad e. & erit $IE = ex : d$. Hanc duc in $\frac{1}{2}$ AI ſive $\frac{1}{2}x$. & fiet area trianguli $AEI = ex^2 : 2d$. Sed ob æquales angulos & latera, Triangula AEI & AEK ſunt æqualia; Ergo trapezium IK $=$ (2 triang. AEI) $= exx : d$. Simili modo pone $BL : LF = d : f$, & $CN : NG = d : g$. & $DP : PH = d : h$. (nam illæ etiam rationes dantur ex datis angulis B, C ac D) & habebitur trapezium $LM = fxx : d$, $NO = gxx : d$ & $PQ = hx^2 : d$. Quamobrem $(ex^2 + fx^2 + gx^2 + hx^2) : d$, ſive $px^2 : d$, ſcribendo p pro $e + f + g + h$ erit æquale trapeziis quatuor $IK + LM + NO + PQ$ & proinde $bx + px^2 : d = aa$ & $x = (-bd + \sqrt{bbdd + 4aadp}) : 2p$.

PROB.

PROB. XX.

A dato puncto C *rectam lineam* CF *ducere quæ cum aliis duabus poſitione datis rectis* AE *&* AF *triangulum datæ magnitudinis* AEF *comprehendet?* TAB. I. FIG. 11.

Sit $AD = a$, $CB = b$, $AEF = cc$, $AF = x$.

Age CD parallelam AE & CB ac EG perpendiculares in AF, & ob proportionales DF : AF ($=$ DC : AE) $=$ CB : EG, hoc eſt, $a + x : x = b : (bx : (a + x)$ erit $bx : (a + x) =$ EG. Hanc duc in $\frac{1}{2}$ AF & emerget $bx^2 : (2a + 2x) = c^2$, adeoque $x = (c^2 + c\sqrt{cc + 2ab}) : b$.

Nihil ſecus recta per datum punctum ducitur quæ triangulum vel trapezium quodvis in data ratione ſecabit.

PROB. XXI.

Punctum C *in data recta linea* DF *determinare, a quo ad alia duo poſitione data puncta* A *&* B *ductæ rectæ* AC *&* BC *datam habeant differentiam?* TAB. I. FIG. 12.

A datis punctis ad datam rectam demitte perpendiculares AD & BF.

Dic AD $= \sqrt{a}$ | Erit AC $= \sqrt{x+a}$, FC $=$
BF $= \sqrt{b}$ | $\sqrt{x} - \sqrt{c}$ & BC $= \sqrt{b + x - 2\sqrt{cx} + c}$.
DF $= \sqrt{c}$ | Sed AC−BC $= \sqrt{d} = \sqrt{x+a}$
AB--BC $= \sqrt{d}$ | $- \sqrt{b + x - 2\sqrt{cx} + c}$.
DC $= \sqrt{x}$

Ergo $b + x - 2\sqrt{cx} + c = x + a - 2\sqrt{dx + ad} + d$, in qua ſcripto $2e$ pro datis $a + d - b - c$ fiet $e + \sqrt{cx} = \sqrt{ad + dx}$ & reducendo $\sqrt{x} = (e\sqrt{c} + \sqrt{de^2 - ad^2 + acd}) : (d - c)$.

Haud ſecus problema reſolvetur ſi linearum AC & BC ſumma, vel quadratorum ſumma aut differentia, vel proportio, vel rectangulum, vel angulus ab iis comprehenſus, detur; vel etiam ſi vice rectæ DC circumferentia circuli, aut alia quævis curva linea adhibeatur, modo calculus (in hoc ultimo præſertim caſu) referatur ad lineam conjungentem puncta A & B.

PROB. XXII.

TAB. I. FIG. 13. *Datis poſitione tribus rectis* AD, AE, BF, *quartam* DF *ducere, cujus partes* DE, EF, *prioribus interceptæ, datarum erunt longitudinum?*

Ad BF demitte perpendicularem EG ut & obliquam EC parallelam AD, rectis

ctis tribus poſitione datis concurrentibus in A, B, & H.

Dic $AB = \sqrt{a}$, $BH = \sqrt{b}$, $AH = \sqrt{c}$, $ED = \sqrt{d}$, $EF = \sqrt{e}$, $HE = \sqrt{x}$

Et ob ſimilia triang. ABH, ECH, erit $AH : AB = HE : EC = \sqrt{ax} : \sqrt{c}$. & $AH : HB = HE : CH = \sqrt{bx} : \sqrt{c}$. Adde HB fietque $CB = (\sqrt{bx} + \sqrt{bc}) : \sqrt{c}$. Inſuper ob ſimilia tr. FEC, FDB eſt $ED : CB = EF : CF = (\sqrt{ebx} + \sqrt{ebc}) : \sqrt{dc}$. Denique (per 12 & 13. II. Elem.) eſt $(ECq - EFq) : 2FC, + \frac{1}{2} FC = CG = (HEq - ECq) : 2CH, - \frac{1}{2} CH$, hoc eſt $(\frac{ax}{c} - e) : (\frac{2\sqrt{ebx} + 2\sqrt{ebc}}{\sqrt{dc}}) + (\sqrt{ebx} + \sqrt{ebc}) : 2\sqrt{dc} = (x - \frac{ax}{c}) : 2\sqrt{\frac{bx}{c}}, - \frac{1}{2}\sqrt{\frac{bx}{c}}$. Sive $(ax - ec)\sqrt{d} : (\sqrt{ebx} + \sqrt{ebc}), + (\sqrt{ebx} + \sqrt{ebc}) : \sqrt{d} = \frac{c\sqrt{x} - a\sqrt{x} - b\sqrt{x}}{\sqrt{b}}$. Hic abbreviandi cauſa, pro $(c - a - b) : \sqrt{b}, - \sqrt{eb} : \sqrt{d}$ ſcribe $\sqrt{m}$ & erit $(ax - ec)\sqrt{d} : (\sqrt{ebx} + \sqrt{ebc}) + \sqrt{ebc}) : \sqrt{d} = \sqrt{mx}$, ac terminis omnibus ductis in $\sqrt{x} + \sqrt{c}$, fiet $(ax - ec)\sqrt{\frac{d}{eb}} + \sqrt{\frac{ebc}{d}}x + c\sqrt{\frac{eb}{d}} = x\sqrt{m} + \sqrt{mcx}$. Iterum

rum pro $a\sqrt{\frac{d}{eb}} - \sqrt{m}$ ſcribe $\sqrt{p}$, pro $\sqrt{mc}$- $\sqrt{\frac{ebc}{d}}$ ſcribe $2\sqrt{pq}$, & pro $-c\sqrt{\frac{eh}{d}} + ec\sqrt{\frac{d}{eb}}$ ſcribe $r\sqrt{p}$, & evadet $x\sqrt{p} = 2\sqrt{pq}\,x + r\sqrt{p}$. & $\sqrt{x} = \sqrt{q} \pm \sqrt{q+r}$. Invento $\sqrt{x}$ ſive HE, age EC parallelam AB & cape FC : BC $= \sqrt{e} : \sqrt{d}$ & acta FED conditionibus quæſtionis ſatisfaciet.

PROB. XXIII.

TAB. I. FIG. 14. *Punctum* Z *determinare a quo ad quatuor poſitione datas rectas lineas* FA, EB, FC, GD, *ſi aliæ quatuor lineæ* ZA, ZB, ZC *&* ZD *in datis angulis ducantur, duarum è ductis* ZA *&* ZB *rectangulum & aliarum duarum* ZC *&* ZD *ſumma detur:*

E lineis elige aliquam poſitione datam FA, ut & poſitione non datam ZA quæ ad illam ducitur, ex quarum longitudinibus punctum Z determinetur, & cæteras poſitione datas lineas produc, donec his ſi opus eſt etiam productis, occurrant, ut vides.

Sit EF $= \sqrt{a}$	Ob ang. tr. AEH datos,
EG $= \sqrt{b}$	datur ratio AE ad AH
EA $= \sqrt{x}$	quam dic. $\sqrt{p}$ ad $\sqrt{q}$ & erit
AZ $= \sqrt{y}$	AH $= \sqrt{\frac{q}{p}}\,x$ & AZ + AH

=

$= ZH = \sqrt{y} + \sqrt{\frac{q}{p}x}$. Sed ob ang. tr. HZB datos, pono $HZ : BZ = \sqrt{n} : \sqrt{p}$. Ergo $ZB = (\sqrt{py} + \sqrt{qy}) : \sqrt{n}$.

Inſuper $AF = \sqrt{a-x}$. Ergo ob ang. tr. AFI datos, pono $AF : AI = \sqrt{p} : \sqrt{r}$ & eſt $AI = (\sqrt{ra} - \sqrt{rx}) : \sqrt{p}$ & $AZ - AI = IZ = \sqrt{y} - (\sqrt{ra} - \sqrt{rx}) : \sqrt{p}$. & ob ang. tr. ICZ datos eſt $IC : ZC = \sqrt{m} : \sqrt{p}$. Ergo $ZC = (\sqrt{py} - \sqrt{ra} + \sqrt{rx}) : \sqrt{m}$.

Ad eundem modum poſito $EG = \sqrt{b}$ & $AG : AK = \sqrt{l} : \sqrt{s}$ & $ZK : ZD = \sqrt{p} : \sqrt{l}$ obtinebitur $ZD = (\sqrt{sb} - \sqrt{sx} - \sqrt{ly}) : \sqrt{p}$.

Sed ex ſtatu quæſtionis pono $ZC + ZD = \sqrt{f}$ & $ZC \times ZD = g$. Erit $(\sqrt{py} - \sqrt{ra} + \sqrt{rx}) : \sqrt{m}, + (\sqrt{sb} - \sqrt{sx} - \sqrt{ly}) : \sqrt{p} = \sqrt{f}$.

Et $y\sqrt{\frac{p}{n}} + \sqrt{\frac{q}{n}}xy = g$, in qua $\sqrt{x} = (g\sqrt{n} - y\sqrt{p}) : \sqrt{qy}$, quo valore ſubſtituto pro $\sqrt{x}$ in æquatione præcedenti, evadet $y\sqrt{\frac{p}{m}} - y\sqrt{\frac{pr}{mq}} - y\sqrt{\frac{l}{p}} + y\sqrt{\frac{s}{q}} = \sqrt{fy} + \sqrt{\frac{ra}{m}}y + \sqrt{\frac{sb}{p}}y + g\sqrt{\frac{sn}{pq}} - g\sqrt{\frac{nr}{mq}}$. Et abbrev. cauſa ſcripto $2\sqrt{b}$ pro $(\sqrt{f} + \sqrt{\frac{ra}{m}} + \sqrt{\frac{sb}{p}}) : (\sqrt{\frac{p}{m}} - \sqrt{\frac{pr}{mq}} - \sqrt{\frac{l}{p}} + \sqrt{\frac{s}{q}})$, &

k pro $(g\sqrt{\frac{sn}{pq}} - g\sqrt{\frac{nr}{mq}}) : (\sqrt{\frac{p}{m}} - \sqrt{\frac{pr}{mq}} - \sqrt{\frac{l}{p}} + \sqrt{\frac{s}{q}})$ fiet $y = 2\sqrt{by} + k$ & $\sqrt{y} = \sqrt{b} \pm \sqrt{b+k}$. Cujus æquationis ope, cum alia $\sqrt{x} = (g\sqrt{n} - y\sqrt{p}) : \sqrt{qq}$ detegitur $\sqrt{x}$. Quod ſufficit ad determinandum punctum Z.

Ad eundem fere modum punctum determinatur, a quo ad plures vel pauciores poſitione datas rectas, totidem aliæ rectæ ducantur ea lege, ut aliquarum ſumma vel differentia vel contentum detur, aut æquetur ſummæ vel differentiæ vel contento, vel ut alias quaſlibet habeant aſſignatas conditiones.

PROBLEMA XXIV.

TAB. I. FIG. 15.

Angulum rectum EAF data EF ſubtendere, quæ tranſibit per datum punctum C, a lineis rectum angulum comprehendentibus æquidiſtans?

Quadratura ABCD compleatur & linea EF biſecetur in G.

Dic CB vel CD $= \sqrt{a}$	Erit CE $= \sqrt{x} - \sqrt{b}$
EG vel FG $= \sqrt{b}$	& CF $= \sqrt{x} + \sqrt{b}$.
CG $= \sqrt{x}$	Dein cum CFq −

BCq

$BCq = BFq$, erit $BF = \sqrt{(x + 2\sqrt{bx} + b - a)}$. Tandem ob ſim. tr. CDE, FBC eſt CE: CD = CF: BF, ſeu $(\sqrt{x} - \sqrt{b}) : \sqrt{a} = (\sqrt{x} + \sqrt{b}) : \sqrt{(x + 2\sqrt{bx} + b - a)}$. Unde $\sqrt{ax} + \sqrt{ab} = (\sqrt{x} - \sqrt{b}) \times \sqrt{(x + 2\sqrt{bx} + b - a)}$. Et $ax + aa + 2b\sqrt{bx} = (x + b - 2\sqrt{bx}) \times (x + 2\sqrt{bx} + b - a)$. Ergo $\sqrt{x} = \sqrt{a + b + \sqrt{aa + 4ab}}$. CG ſic inventa dat CE vel CF, quæ determinando punctum E vel F problemati ſatisfacit.

PROB. XXV.

Ad circulum centro C, radio DC deſcriptum ducere Tangentem DB, cujus pars PB inter rectas poſitione datas AP, AB ſita, ſit datæ longitudinis? TAB. II. FIG. 1.

A centro C ad alterutram rectarum poſitione datarum, puta AB, demitte normalem CE, hancque produc donec tangenti DB occurrat in H. Ad eandem AB demitte etiam normalem PG.

Sit $EA = \sqrt{a}$	Ob ſim. tr. PGB, CDH
$EC = \sqrt{b}$	eſt $\sqrt{(d - x)} : x = \sqrt{c} :$
$ED = \sqrt{c}$	CH. Ergo $CH = \sqrt{cd} : \sqrt{(d - x)}$ & $\sqrt{cd} : \sqrt{(d - x)}$
$BP = \sqrt{d}$	$+ \sqrt{b} = EH$.
$PG = \sqrt{x}$	

Sed $\sqrt{x} : \sqrt{(d - x)} = \sqrt{(d - x)} + \sqrt{b}$:

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$EB = \sqrt{\left(\frac{bd}{x} - b\right)} + \sqrt{\frac{cd}{x}}$ & ob ang. PAB datum $\sqrt{x} : AG = \sqrt{e} : \sqrt{f}$. Ergo $AG = \sqrt{f}\,\frac{x}{e}$ & rurſus $EB = EA + AG + BG = \sqrt{a} + \sqrt{\frac{fx}{e}} + \sqrt{(d - x)}$. Et $\sqrt{a} + \sqrt{\frac{fx}{e}} + \sqrt{(d-x)} = \sqrt{\left(\frac{bd}{x} - b\right)} + \sqrt{\frac{cd}{x}}$.

Et $a + \frac{fx}{e} + \frac{cd}{x} + 2\sqrt{\frac{af}{e}}\,x - 2\sqrt{\frac{acd}{x}} - 2\sqrt{\frac{edf}{e}} = \frac{bd}{x} - b - 2d\sqrt{\frac{b}{x}} + 2\sqrt{bx} + d - x$. atque multiplicatis per ex & ordinatis terminis fit.

$$x^2 \left(\begin{array}{llll} & +dex & & \\ +2\sqrt{aefx^3} & +2x\sqrt{cdfe} & +2e\sqrt{acdx} & +bde \\ -2e\sqrt{bx^3} & -2aex & -2ed\sqrt{bx} & -cde \\ & -bex & & \end{array}\right)$$

$: (e+f) = 0.$

PROB. XXVI.

TAB. II. FIG. 2. *Invenire punctum* D, *a quo tres rectæ* DA, DB, DC, *ad totidem alias* poſitione *datas rectas* AE, BF, CF *perpendiculariter demiſſæ, datam inter ſe rationem obtineant?*

E rectis poſitione datis produc unam; puta BF ut & ejus perp. BD donec reliquis AE & CF occurrant; BF quidem in E & F, BD autem in H & G.

Dic $EB = \sqrt{x}$ $EF = \sqrt{a}$ $BD = \sqrt{y}$	Erit $BF = \sqrt{a} - \sqrt{x}$, & cum ob poſitionem datam rectarum EF, EA & FC

anguli E & F, adeoque rationes laterum tr. EBH & FBG dentur, ſit $EB : BH = \sqrt{d} : \sqrt{e}$. Erit $BH = \sqrt{\frac{ex}{d}}$ & $EH = \sqrt{(EBq + BHq)} = \sqrt{\left(\frac{ex}{d} + x.\right)}$ Sit etiam $BF : BG = \sqrt{d} : \sqrt{f}$. eritque $BG = \sqrt{\frac{fa}{d}} - \sqrt{\frac{fx}{d}}$ & $FG = \sqrt{(BFq + BGq)} = \left(\sqrt{\frac{af}{d}} - \sqrt{\frac{fx}{d}}\right) \times \sqrt{d+f}$. Præterea $HD = \sqrt{\frac{e}{d}}\, x - \sqrt{y}$ & $GD = \sqrt{\frac{fa}{d}} - \sqrt{\frac{fx}{d}} - y$, adeoque, ob $AD : HD\ (= EB : EH) = \sqrt{x} : \sqrt{\left(\frac{ex}{d} + x\right)}$ & $DC : DG\ (= BF : FG) = (\sqrt{a} - \sqrt{x}) : \left(\sqrt{\frac{af}{d}} - \sqrt{\frac{fx}{d}}\right) \times \sqrt{d+f}$, ſit $AD = (\sqrt{ex}$

 $-\sqrt{dy})$

$-\sqrt{dy}$) : $\sqrt{e+d}$ & DC = ($\sqrt{af}-\sqrt{fx}-\sqrt{dy}$) : $\sqrt{d+f}$. Tandem ob datas rationes linearum BD, AD, DC, ſit BD : AD = ($\sqrt{d+e}$) : $\sqrt{h}-\sqrt{d}$ & erit ($\sqrt{hy}-\sqrt{dy}$) : ($\sqrt{e+d}$) = AD = ($\sqrt{ex}-\sqrt{dy}$) : ($\sqrt{e+d}$) ſeu $\sqrt{hy}=\sqrt{ex}$. Sit etiam BD : DC = ($\sqrt{d+f}$) : ($\sqrt{k}-\sqrt{d}$) eritque ($\sqrt{ky}-\sqrt{dy}$) : $\sqrt{d+f}$ = DC = ($\sqrt{fa}-\sqrt{fx}-\sqrt{dy}$) : $\sqrt{d+f}$ ſeu $\sqrt{ky}=\sqrt{fa}-\sqrt{fx}$.

Eſt ergo $\sqrt{\frac{ex}{h}}$ (= $\sqrt{y}$) = ($\sqrt{fa}-\sqrt{fx}$) : $\sqrt{k}$ & reducendo $\sqrt{fha}$: ($\sqrt{ek}+\sqrt{fh}$) = $\sqrt{x}$. Quare cape EB : EF = $\sqrt{h}$: ($\sqrt{\frac{e}{f}k}+\sqrt{h}$); dein BD : EB = $\sqrt{e}$: $\sqrt{b}$ & habebitur punctum quæſitum D.

PROB. XXVII.

TAB. II. FIG. 3. *Invenire punctum* D, *a quo tres rectæ* DA, DB, DC *ad data tria puncta* A, B, C, *ductæ, datam inter ſe rationem obtineant?*

E datis tribus punctis junge duo quævis, puta A & C, & à tertio B ad lineam conjungentem AC demitte perp. BE.

Dic

Dic $AE = \sqrt{a}$ | Erit $ADq = x + y$ & $FCq = \sqrt{b} - \sqrt{x}$, & $CDq = FCq + FDq = b - 2\sqrt{bx} + x + y$ & $EF = \sqrt{x} - \sqrt{a}$. Et (ob $EFq + \overline{EBq + FDq}$ Quadr.) $= BDq = x - 2\sqrt{ax} + a + c + 2\sqrt{cy} + y$.

$AC = \sqrt{b}$

$EB = \sqrt{c}$

$AF = \sqrt{x}$

$FD = \sqrt{y}$

Jam cum ſit AD ac CD in data ratione, ſit iſta ratio $\sqrt{d}$ ad $\sqrt{e}$ & erit $CD = \sqrt{(\frac{e}{d}x + \frac{e}{d}y)}$. Atque etiam (ob $AD : BD = \sqrt{d} : \sqrt{f}$) erit $BD = \sqrt{(\frac{f}{d}x + \frac{f}{d}y)}$. Ergo $\frac{e}{d}x + \frac{e}{d}y$ ($= CDq$) $= x - 2\sqrt{bx} + b + y$ & $\frac{f}{d}x + \frac{f}{d}y = (BDq) = x - 2\sqrt{ax} + a + c + 2\sqrt{cy} + y$. In priori pro $(d-e) : d$ ſcribo $\sqrt{p}$ & in poſteriori $\sqrt{q}$ pro $(d-f) : d$, oriturque $x\sqrt{p} = 2\sqrt{bx} - b - y\sqrt{p}$ & $x\sqrt{q} = 2\sqrt{ax} - a - c - 2\sqrt{cy} - y\sqrt{q}$. Et inventis $\sqrt{x}$ & $\sqrt{y}$, hoc eſt, AF & FD, determinatur punctum quæſitum D.

PROB. XXVIII.

Rectam DC *datæ longitudinis in datam Conicam ſectionem* DAC *ſic inſcribere, ut ea per punctum* G *poſitione datum tranſeat?* TAB. II. FIG. 4.

Sit AF axis curvæ, & a punctis D, G & C ad hanc demitte normales DH, GE, & CB. jam ad determinandam poſitionem rectæ DC, puncti D aut C inventio proponi poteſt; ſed cum hæc ſint germana & adeo paria, ut ad alterutrum determinandum operatio ſimilis evaſura eſſet, ſive quærerem CG, CB aut AB, ſive comparia DG, DH, aut AH. Ea propter (quæ regula eſt electionis terminorum ad ineundum calculum) de tertio aliquo puncto proſpicio quod utrumque D & C ſimiliter reſpectet & una determinet. Et video hujusmodi eſſe punctum F.

Sit $AE = \sqrt{a}$, $EG = \sqrt{b}$, $DC = \sqrt{c}$, $EF = \sqrt{z}$.

Cum relatio inter AB & BC habeatur in æquatione quam ſuppono pro Conica ſectione determinanda datam eſſe, ſit $AB = \sqrt{x}$ & $BC = \sqrt{y}$ & erit $FB = \sqrt{x} - \sqrt{a} - \sqrt{z}$. Et propter $CE : EF = CB : FB$, erit iterum $FB = \sqrt{yz} : \sqrt{b}$. Ergo $\sqrt{x} - \sqrt{a} + \sqrt{z} = \sqrt{yz} : \sqrt{b}$.

His ita præparatis tolle $\sqrt{x}$ per æquationem quæ curvam deſignat. Ut ſi curva ſit parabola, per æquationem $\sqrt{rx} = y$ deſignata, ſcribe $y : \sqrt{r}$ pro $\sqrt{x}$ & orietur $y\sqrt{r} - \sqrt{a} + \sqrt{z} = \sqrt{yz} : \sqrt{b}$. Et ex-

tracta

tracta radice $\sqrt{y} = \frac{1}{2}\sqrt{\frac{rz}{b}} \pm \sqrt{\frac{rz}{4b} + \sqrt{ar} - \sqrt{rz}}$ unde (14.) patet $\sqrt{\frac{rz}{b} + 4\sqrt{ar} - 4\sqrt{rz}}$ = BC−DH ſeu eſſe differentiam gemini valoris $\sqrt{y}$, adeoque (demiſſo DK in CB normali) valere CK. Eſt autem FG : GE = DC : CK, hoc eſt, $\sqrt{b+z} : \sqrt{b} = \sqrt{c} : \sqrt{(\frac{rz}{b} + 4\sqrt{ar} - 4\sqrt{rz})}$. Ergo $z^2 = (4b\sqrt{rz^3} - brz - 4bz\sqrt{ar} + 4bb\sqrt{rz} - 4b^2\sqrt{ar} + cb^2) : r$, æquatio quatuor tantum dimenſionum (ut patet poſito z pro $\sqrt{z}$) quæ ad octo aſcendiſſet, ſi quæſiviſſem CG vel AB vel CB.

PROB. XXIX.

Datum angulum per datum numerum multiplicare, vel dividere? TAB. II. FIG. 5.

In angulo quovis FAG inſcribe lineas AB, BC, CD, DE &c. & ejuſdem cujusvis longitudinis, & erunt tr. ABC, BCD, CDE, DEF &c, Iſocelia: Ergo per 32. I. Elem. erit ang. CBD = ang. A + ACB = 2 ang. A, & ang. DCE = 3. ang. A & ang. EDF = 4. ang. A & ang. FEG = 5. ang. A &c. Poſitis jam AB, BC, CD &c. radiis æqualium circulorum, perp. BK,

 CL,

CL, DM &c demiſſa in AC, BD, CE &c erunt ſinus iſtorum angulorum & AK, BL, CM, DN &c. ſinus complementorum ad rectum. Vel poſita AB diametro, illæ AK, BL, CM &c erunt chordæ. Sit ergo $AB = 2r$ & $AK = x$. Dein ſic operare.

$AB : AK = AC : AL$

ſeu $2r : x = 2x : x^2 : r$

& $AL - AB$) = BL. Duplicatio
ſeu $x^2 : r, - 2r$)

$AB : AK = AD (= 2AL - AB) : AM.$

ſeu $2r : x = (2x^2 : r, -2r) : (x^3 : r^2, -x)$

& $AM - AC$) = CM Triplicatio.
ſeu $x^3 : r^2, -3x$)

$AB : AK = AE (= 2AM - AC) : AN.$

ſeu $2r : x = (2x^3 : r^2, -4x) : (x^4 : r^3, -2x^2 : r)$

& $AN - AD$) = DM Quadruplic.
ſeu $x^4 : r^3, -4x^2 : r, + 2r$)

$AB : AK = AF (2AN - AD) : AO.$

ſeu $2r : x = (2x^4 : r^3, -6x^2 : r, + 2r) : (x^5 : r^4, -3x^3 : r^2, + x)$

ſeu $AO - AE$) = EO Quintuplic.
$x^5 : r^4, -5x^3 : r^2, + 5x$)

Et ſic deinceps. Quod ſi velis angulum in aliquot partes dividere, pone q pro BL, CM, DN &c & habebis $x^2 - 2r^2 = qr$ ad biſectionem, $x^3 - 3r^2 x = qrr$ ad

ad triſect. $x^4 - 4r^2x^2 + 2r^4 = qr^3$ ad quadriſect. $x^5 - 5r^2x^3 + 5r^4x = qr^4$ ad quinquiſect. &c.

Ubi in aliquo caſu particulari dantur in numeris radices r & linea dati anguli complementum ad duos rectos ſubtendens q, ut quod radius ſit 10 & ſubtenſa 3, ſubſtitue numeros illos pro r & q in æquatione, puta, ſectionis anguli in quinque partes æquales, & provenit æquatio numeralis $x^5 - 500x^3 + 50000x - 30000 = 0$, cujus radix tandem extracta erit x ſeu linea complementum quintæ partis anguli illius dati ſubtendens.

PROB. XXX.

Cometæ in linea recta BD uniformiter TAB. II. FIG. 6.
progredientis poſitionem curſus ex tribus obſervationibus determinare?

Sit A occulus ſpectatoris, B locus cometæ in prima obſervatione, C in ſecunda ac D in tertia; quærenda erit inclinatio lineæ BD ad lineam AB. Ex obſervationibus itaque dantur anguli BAC BAD, adeoque ſi BH ducatur ad AB normalis & occurrens AC & AD in E & F, ex aſſumpto utcumque AB dabuntur BE & BF,

tan-

tangentes nempe præfatorum angulorum, reſpectu radii AB.

Eſto AB $= a$	Ex datis obſervationum
BE $= b$	intervallis dabitur ratio
BF $= c$	BC ad BD quæ ſi ponatur

b ad e & agatur DG parallela AC, cum ſit etiam BE : BG $= b : e$ & BE dicta fuerit b, erit BG $= e$, adeoque GF $= e - c$. Adhæc ſi demittatur DH normalis ad BG, ob tr. ABF & DHF ſimilia & ſimiliter ſecta lineis AE ac DG, erit FE : AB $=$ FG : HD, hoc eſt $c - b : a = e - c : (ae - ac) : (c - b) =$ HD. Erit etiam FE : FB $=$ FG : FH, hoc eſt $c - b : c = e - c : (ce - cc) : (c - b) =$ FH; cui adde BF $= c$, fitque BH $= (ce - cb) : (c - b)$. Quare eſt $(ce - cb) : (c - b)$ ad $(aa - ac) : (c - b)$ (ſive $ce - cb$ ad $ae - ac$, vel $(ce - cb : (e - c)$ ad a) ut BH ad HD; hoc eſt ut tangens anguli HDB ſive ABK ad radium. Quare cum a ſupponatur eſſe radius, erit $(ce - cb) : (e - c)$ tangens anguli ABK, adeoque facta reſolutione, erit ut $e - c$ ad $e - b$ (ſive GF ad GE) ita c (ſive tangens anguli BAF) ad tangentem anguli ABK.

Dic ergo, ut tempus inter primam & ſecundam obſervationem ad tempus inter primam ac tertiam, ita tangens anguli BAE ad quartam proport. N. Dein

ut

ut differentia inter N & tangentem anguli BAF ad diff. inter N & tangent. ang. BAE, ita tangens ang. BAF ad tang. anguli ABK.

PROP. XXXI.

Radiis a puncto lucido ad ſphæricam ſuperficiem refringentem divergentibus, invenire concurſus ſingulorum refractorum cum axe ſphæræ per punctum illud lucidum tranſeunte? TAB. II. FIG. 7.

Sit A punctum illud lucidum & BV Sphæra, cujus axis AD, centrum C & vertex V: ſitque AB radius incidens & BD refractus ejus, ac demiſſis ad radios iſtos perp. CE & CF, ut & BG perp. ad AD, agatur BC.

Dic $AC = \sqrt{a}$	Erit $AB = \sqrt{(a \cdot 2\sqrt{ax} + r)}$.
VC vel $BC = \sqrt{r}$	Nam $AG = \sqrt{a} - \sqrt{x}$ &
$CG = \sqrt{x}$	$BG = \sqrt{(r - x)}$.
$CD = \sqrt{z}$	

Et $CE = \sqrt{ar - qx} : \sqrt{a - 2\sqrt{ax} + r}$, ob ſim. tr. ABG & ACE.

Item $GD = \sqrt{z} + \sqrt{x}$, BD $(= \sqrt{GDq - BGq})$ $= \sqrt{(z + 2\sqrt{zx} + r)}$.

Et $CF = \sqrt{(rz - xz)} : \sqrt{(z - 2\sqrt{xz} + r)}$. ob ſim. tr. DBG & DCF.

Præterea cum ratio ſinuum incid. & refract, adeoque CE ad CF detur, hanc

pone

pone $\surd a$ ad $\surd f$, & erit $\surd(ar-ax)$: $\surd(a-2\surd ax+r)$, :: CF $=\surd a : \surd f$, ſeu $\surd(r-x) : \surd(a-2\surd ax+r)$, :: CF $=\surd 1 : \surd f$. Ergo rurſus $\surd(rf-fx) : \surd(a-2\surd ax+r) =$ CF $=\surd(rz-xz) : \surd(z-2\surd xz+r)$. Et $f : (a-2\surd ax+r) = z : (z-2\surd zx+r)$, adeoque $\surd z = (f\surd x \pm \surd\overline{ffx-bfr}) : (2\surd ax+f-a-r) = b$ Inventum eſt itaque $\surd z$ hoc eſt longitudo CD, ergo & punctum quæſitum D quo refractus BD concurrit cum axe. Q. E. F.

Poſui hic incidentes radios divergentes eſſe & in medium denſius incidere; ſed mutatis mutandis problema perinde reſolvitur ubi convergunt vel incidunt è denſiori medio in rarius.

PROB. XXXII.

TAB. II. FIG. 9, 11. *Si Conus plano quolibet ſecetur, invenire figuram ſectionis?*

Sit ABC conus circulari baſi BC inſiſtens; IEM ejus ſectio quæſita; KILM alia quælibet ſectio parallela baſi & occurrens priori in HI; & ABC tertia ſectio perpendiculariter biſecans priores duas in EH & KL & conum in triangulo ABC. Et producto EH donec occurrat ipſi AK in

in D, agantur EF ac DG parallelæ KL & occurrentes AB & AC in F ac G.

Dic $EF = a$	Ob ſim. tr. EHL, EDG,
$DG = b$	eſt ED : DG = EH : HL
$ED = c$	$= bx : c$.
$EH = x$	Ob ſim. tr. DEF, DHK
$HI = y$	eſt DE : EF = DH ($= c$

$- x$ in fig. 9. & $c + x$ in fig. 11.); $HK = (ac \pm ax) : c$. Denique cum ſectio KIL ſit parallela baſi, adeoque circularis, erit $HK \times HL = HIq$, hoc eſt $abx : c \mp abx^2 : c^2 = y^2$ æquatio quæ exprimit relationem inter x & y hoc eſt inter axem & ordinatim applicatam ſectionis EIM, quæ æquatio cum ſit ad Ellipſin in fig. 9 & ad Hyperbolam in fig. 11, patet ſectionem illam perinde Ellipticam vel Hyperbolicam eſſe.

Quod ſi ED nullibi occurrat AK ipſi parallela exiſtens, tunc erit $HK = EF = a$ & inde $abx : c$ ($= HK \times HL$) $= y^2$ æquatio ad parabolam.

PROB. XXXIII.

Si recta XY *circa axim* AB, *ad diſtantiam* CD, *in data inclinatione ad planum* DCB *convolvatur & ſolidum* PQRUTS *iſta convolutione generatum, ſecetur plano quolibet* INQLK, *invenire figuram ſectionis?* TAB. II. FIG. 2.

Eſto

Eſto BHQ vel GHO inclinatio axis AB ad planum ſectionis; & L quilibet concurſus rectæ XY cum plano illo. Age DF parallelam AB & ad AB, DF & HO demitte perpend. LG, LF & LM ac junge FG & MG.

Dic CD $= \sqrt{a}$
CH $= \sqrt{b}$
HM $= \sqrt{x}$
ML $= \sqrt{y}$

Ob ang. datum GHO, poſito MH : HG $= \sqrt{d}$; $\sqrt{e}$, erit $\sqrt{ex} : \sqrt{d} =$ GH & $\sqrt{b} + \sqrt{ex} : \sqrt{d}$ $=$ GC vel FD. Adhæc ob ang. datum LDF, (nempe inclinationem rectæ XY ad planum GCDF) poſito FD : FL $= \sqrt{g}$: $\sqrt{h}$ erit $\sqrt{bb} : \sqrt{g} + \sqrt{hex} : \sqrt{dg} =$ FL, cujus quadrato adde FGq ($=$ DC$q = a$) & emerget GL$q = a + (bdh + 2h\sqrt{bdex} + ehx) : dg$. Hinc aufer MG$q$ ($=$ HMq $-$ HGq ſeu $x - ex : d$) & reſtabit $(adg + bdh + 2h\sqrt{bdex} + hex - dgx + egx) : dg$ ($=$ MLq) $= y$ æquatio quæ exprimit relationem inter $\sqrt{x}$ & $\sqrt{y}$, hoc eſt inter HM axem ſectionis & ML ordinatim applicatam: in qua proinde cum $\sqrt{y}$ & $\sqrt{x}$ ad duas tantum dimenſiones aſcendant, patet figuram INQLK eſſe conicam ſectionem. Utpote ſi ang. MHG major ſit ang. LDF, Ellipſis erit hæc figura; ſi minor, Hyperbola; ſi æqualis, vel Parabola, vel coincidentibus inſuper punctis (&H) parallelogrammum.

PROB.

PROB. XXXIV.

Sit ad AF erigatur perpend. AD datæ longitudinis & normæ DEF crus unum ED continuo tranſeat per punctum D, dum alterum crus EF æquale AD dilabatur ſuper AF; invenire curvam HIC quam crus EF medio ejus puncto C deſcribit? TAB. II. FIG. 10.

Sit BC vel CF $= \sqrt{a}$, perp. CB $= \sqrt{y}$, AB $= \sqrt{x}$. Ob ſim. tr. FBC, FEG erit BF ($= \sqrt{a-y}$) : BC + CF ($= \sqrt{y} + \sqrt{a}$) $=$ EF ($= 2\sqrt{a}$) : EG + GF $=$ (AG + GF) $=$ AF. Quare $(2\sqrt{ay} + 2a) : \sqrt{(a-y)} =$ AF $=$ (AB + BF) $= \sqrt{x} + \sqrt{(a-y)}$. Et $2\sqrt{ay} + 2a = \sqrt{x} \times \sqrt{(a-y)} + a - y$.

Ergo $(y + 2\sqrt{ay} + a) : \sqrt{x} = \sqrt{(a-y)}$ & $(\sqrt{a} + \sqrt{y}) : \sqrt{x} = \sqrt{a-y} : (\sqrt{a} + \sqrt{y})$ atque quadratis partibus & diviſis per $\sqrt{a} + \sqrt{y}$ (25.) prodit $y\sqrt{y} + 3y\sqrt{a} + 3a\sqrt{y} + x\sqrt{y} + \sqrt{a^3} - x\sqrt{a} = 0$.

Idem Aliter.

TAB. III. FIG. 1.

In BC cape hinc inde BI & CK æquales CF, & age KF, HI, HC ad DF, quarum HC ad DF occurrant ipſis AF &

IK in M & N, & in HC demitte normalem IL. Eritque ang K = $\frac{1}{2}$ BCF = $\frac{1}{2}$ EGF = GFD = AMH = MHI = CIL, adeoque tr. rectangula KBF, FBN, HLI & ILC ſimilia.

Sit FC = a HI = x IC = y	Erit ergo BN ($= 2a - y$) : BK ($= y$) = LC : LH = CIq ($= yy$) : HIq ($= x^2$).

Ergo $2ax^2 - yx^2 = y^3$. Ex qua æquatione facile colligitur hanc curvam eſſe Ciſſoidem veterum, ad circulum cujus centrum ſit A ac radius AH, pertinentem.

PROB. XXXV.

TAB. III. FIG. 2. *Si datæ longitudinis recta* ED *angulum datum* EAD *ſubtendens ita moveatur ut termini ejus* D & E *anguli iſtius latera* AD & AE *perpetim contingant; proponatur curvam* FCG *determinare quam punctum quodvis* C *in recta iſta* ED *datum deſcribit?*

A dato puncto C age CB parallelam EA.

Dic AB = $\sqrt{x}$ BC = $\sqrt{y}$ CE = $\sqrt{a}$ CD = $\sqrt{b}$	Ob ſim. tr. DCB, DEA, erit EC : AB = CD : BD = $\sqrt{bx} : \sqrt{a}$. Dein demiſſo perp. CH,

ob

ob datum ang. DAE vel DBC, adeoque ob datam rationem laterum tr. rectanguli BCH, ſit $\sqrt{a} : \sqrt{e} = BC : BH$, eritque $BH = \sqrt{bx} : \sqrt{a}$. Et $BD - BH = HD = (\sqrt{bx} - \sqrt{ey}) : \sqrt{a}$. Jam in tr. BCH, ob ang rectum BHC eſt $BCq - BHq = CHq = y, -ey : a$. item in tr. CDH, ob ang. CHD rectum, eſt $CDq - CHq = HDq$, hoc eſt $b - y + ey : a = (\sqrt{bx} - \sqrt{ey}) : \sqrt{a}$ quadrato $= (bx - 2\sqrt{bexy} + ey) : a = (HDq) = (ab - ay + ey) : a$. & per reductionem $y = (2\sqrt{bexy} + ab - bx) : a$; ubi cum incognitæ quantitates ſint duarum tantum dimenſionum, patet curvam eſſe conicam ſectionem. Præterea fit $\sqrt{y} = (\sqrt{bex} \pm \sqrt{bex - abx + a^2b}) : a$. Ubi in termino radicali coefficiens ipſius x eſt $e - a$. atqui erat $\sqrt{a} : \sqrt{e} = BC : BH$ & $BC > BH$, nempe hypotenuſa tr. rectanguli major latere: Ergo $\sqrt{a} > \sqrt{e}$ & $e - a$ negativa eſt quantitas, adeoque curva erit Ellipſis.

PROB. XXXVI.

Si norma EBD *ita moveatur ut crus unum* ED *continuo ſubtendat ang. rectum* EAB, *dum terminus alterius cruris* BD *deſcribat curvam aliquam lineam* FDG; *invenire lineam iſtam* FDG *quam punctum* D *deſcribit?* TAB. III. FIG. 3.

A puncto D ad latus AC demitte perp. DC.

Dic $AC = \sqrt{x}$
$DC = \sqrt{y}$
$EB = \sqrt{a}$
$BD = \sqrt{b}$

In tr. BDC, ob ang. rectum ad C, eſt $BCq = BDq - DCq = b - y$. Ergo $BC = \sqrt{(b-y)}$ & $AB = \sqrt{x} - \sqrt{(b-y)}$. Dein ob ſim. tr. BEA, DBC, eſt BD : DC = EB : AB $= \sqrt{ay} : \sqrt{b}$. Quare $\sqrt{y} = (\sqrt{abx} \pm b\sqrt{:a+b-x}) : (a+b)$. Unde patet iterum curvam eſſe Ellipſin.

Hæc ita ſe habent ſi ang. EBD & EAB recti ſint; ſi non, dummodo ſint æquales ſic procede. Demitte DC perp. ad AC ut ante & age DH conſtituens angulum HAE, puta obtuſum.

TAB. III. FIG. 4.

Dic $EB = \sqrt{a}$
$BD = \sqrt{b}$
$AH = \sqrt{x}$
$HD = \sqrt{y}$

Ob ſim. tr. EAB, BHD erit BD : DH = EB : AB $= \sqrt{ay} : \sqrt{b}$. AH − AB = BH $= \sqrt{x} - \sqrt{ay} : \sqrt{b}$. Præterea in tr. DHC, ob omnes ang datos, adeoque datam rationem laterum, Eſto DH : HC $= \sqrt{b} : \sqrt{e}$, erit $HC = \sqrt{ey} : \sqrt{b}$ & HB x HC $= (\sqrt{bexy} - y\sqrt{ae}) : b$. Denique per 12. II. Elem. in tr. BHD eſt $BDq = BHq + DHq + 2BH \times HC$, ſeu $b = x - 2\sqrt{axy} : \sqrt{b}, + ay : b + y + (2\sqrt{bexy} - 2y\sqrt{ae}) : b$. Et $\sqrt{x} = (\sqrt{ay} - \sqrt{ey} \pm \sqrt{ey - by + bb}) : \sqrt{b}$. Ubi cum $\sqrt{b} > \sqrt{e}$

hoc

hoc eſt $e-b$ quantitas ſit negativa, patet iterum curvam eſſe Ellipſin.

PROB. XXXVII.

In dato angulo PAB actis utcunque rectis BD, PD in indata ratione, hac ſemper lege, ut BD ſit parallela AP & PD terminetur ad punctum P in recta AP datum; invenire locum puncti D? TAB. III. FIG. 5.

Age CD parallelam AB & DE perpendic. AP.

Dic AP $= \sqrt{a}$, CP $= \sqrt{x}$, CD $= \sqrt{y}$ | Eſto BD : PD $= \sqrt{d} : \sqrt{e}$ & erit AC vel BD $= \sqrt{a} - \sqrt{x}$ & PD $= \sqrt{(ea - ex)} : \sqrt{d}$. Sit dein ob ang. DCE datum CD : CE $= \sqrt{d} : \sqrt{f}$ & erit CE $= \sqrt{fy} : \sqrt{d}$ & EP $= \sqrt{x} - \sqrt{fy} : \sqrt{d}$. Atqui ob ang. ad E rectos eſt CDq − CEq (= EDq) = PDq − EPq. Hoc eſt $y - fy : d = (ea - 2e\sqrt{ax} + ex) : d, - x + 2\sqrt{fxy} : \sqrt{d} - fy : d$. Et $\sqrt{y} = (\sqrt{fx} \pm \sqrt{ea - 2e\sqrt{ax} + fx + ex - dx}) : \sqrt{d}$. Ubi cum $\sqrt{x}$ & $\sqrt{y}$ in æquatione penult. non niſi ad duas dimenſiones aſcendant, locus puncti D erit Conica ſectio, eaque Hyperb. Parab. vel Ellipſis, prout $e - d + f$ (coefficiens ipſius x in æqua-

 tione

tione poſteriori) ſit majus, æquale vel minus nihilo.

PROB. XXXVIII.

TAB. III. FIG. 6.

Rectis duabus VE *&* VC *poſitione datis & ab alia recta* PE *circa polum poſitione datum* P *vertente ſectis utcumque in* C *&* E; *ſi recta intercepta* CE *dividatur in partes* CD, DE, *rationem datam habentes, proponatur invenire locum puncti* D.

Age VP, eique parallelas DA, EB occurrentes VC in A & B.

Dic VP $= \sqrt{a}$ VA $= \sqrt{x}$ AD $= \sqrt{y}$	Cum detur ratio CD ad DE, vel converſe ratio CD ad CE, hoc eſt ratio

DA ad EB, ſit hæc ratio $\sqrt{d}$ ad $\sqrt{e}$ & erit EB $= \sqrt{ey} : \sqrt{d}$. Dein cum detur ang. EVB, adeoque ratio EB ad VB ſit hæc ratio $\sqrt{e}$ ad $\sqrt{f}$; & erit VB $= \sqrt{fy} : \sqrt{d}$. Denique ob ſim. tr. CEB, CDA, CPV eſt EB : CB = DA : CA = VP : VC, & *componendo* EB + VP : CB + VC = DA + VP : CA + VC. hoc eſt $(\sqrt{ey} : \sqrt{d}, + \sqrt{a}) : (\sqrt{fy} : \sqrt{d}) = (\sqrt{y} + \sqrt{a}) : \sqrt{x}$. Et $\sqrt{eyx} + \sqrt{dax} = y\sqrt{f} + \sqrt{afy}$. Ubi cum indefinitæ $\sqrt{x}$ & $\sqrt{y}$ non niſi ad duas dimenſio-

ſiones aſcendant ſequitur curvam VD, in qua punctum D perpetim reperitur eſſe conicam ſectionem, eamque Hyperbolam, quia $\surd x$ eſt unius tantum dimenſionis & in termino $\surd exy$ per alteram indefinitam $\surd y$ multiplicatur.

PROBLEMA XXXIX.

Si rectæ duæ AC, BC *a duobus poſitione datis punctis* A & B *in data quavis ratione ad tertium quodvis punctum* C *ducantur; invenire locum puncti concurſus* C? TAB. III. FIG. 7.

Junge AB & ad hanc demitte normalem CD:

Dic AB $= \surd a$	Erit AC $= \surd(x+y)$.
AD $= \surd x$	BD $= \surd a - \surd x$ & BC
DC $= \surd y$	$= \surd(BDq - DCq) = \surd$

$(x - 2\surd ax + a + y)$. Sed AC : BC $= \surd d : \surd e$. Ergo $\surd(ex + ey) : \surd d =$ BC $= \surd(x - 2\surd ax + a + y)$ & $\surd y = \surd d \times \surd(a - 2\surd ax - x)\,;\,(e - d)$. Ubi cum x ſit negativum & ſola unitate affectum & ang. ADC rectus, patet curvam in qua punctum C locatur eſſe circulum. Nempe in recta AB cape puncta E & F, ita ut ſint $\surd d : \surd e =$ AE:BE = AF:BF & erit EF circuli hujus diameter.

Et hinc e converſo patet, hoc theorema, quod in circuli cujuſvis diametro EF infinite producta, datis utcumque duobus punctis A & B hac lege ut ſit AE : AF = BE : BF, & a punctis hiſce actis duabus rectis AC, BC concurrentibus ad circulum in puncto quovis C; erit AC ad BC in data ratione AE ad BE.

PROB. XL.

TAB. III. FIG. 8. *Si punctum lucidum A radios verſus refringentem ſuperficiem planam* CD *ejiciat; invenire radium* AC, *cujus refractus* CB *impinget in datum punctum* B?

A puncto lucido iſto ad refringens planum demitte perp. AD & cum eo utrinque producto concurrat refractus radius BC in E & perpendiculum a puncto B demiſſum in F & agatur BD.

Dic AD $= \sqrt{a}$ DB $= \sqrt{b}$ BF $= \sqrt{c}$ DC $= \sqrt{x}$	Statue rationem ſinuum incedentiæ & refractionis, hoc eſt, ſinuum angulorum CAD,

CED rationem eſſe $\sqrt{d}$ ad $\sqrt{e}$, & cum EC & AC (ut notum eſt) ſint in eadem ratione & AC ſit $\sqrt{(a+x)}$, erit $EC = \sqrt{(ad+dx)}$: $\sqrt{e}$.

$\sqrt{e}$. Dein $ED = \sqrt{(ECq - CDq)} = \sqrt{(ad + dx - ex)} : \sqrt{e}$, & $DF = \sqrt{(b - c)}$, atque $EF = \sqrt{(b - c)} + \sqrt{(ad + dx - ex)} : \sqrt{e}$. Tandem ob ſim. tr. ECD EBF eſt ED : DC = EF : FB. Hoc eſt $\sqrt{\frac{ad + dx - ex}{e}} :$

$\sqrt{x} = (\sqrt{b - c} + \sqrt{\frac{ad + dx - ex}{e}}) : \sqrt{c}$. Sive

$1 : \sqrt{x} = \sqrt{\frac{be - ce}{ad + dx - ex}} + \sqrt{1} : \sqrt{c}$. Adeoque $\sqrt{c} = \sqrt{\frac{bex - cex}{ad + dx - ex}} + \sqrt{x}$ Et $c - 2\sqrt{cx} + x = (bex - cex) : (ad + dx - ex)$. Ergo $x^2 - 2\sqrt{cx^3} + (adx + cdx - bex - 2ad\sqrt{cx} + acd) : (d - e) = 0$.

PROB. XLI.

Invenire locum verticis trianguli D *cujus baſis* AB *datur & anguli ad baſim* DAB, DBA *datam habent differentiam* . TAB. III. FIG. 9.

Acta MPQ conſtituente hinc inde angulos APQ, BPM æquales ſemidifferentiæ angulorum ad baſem, ita ut ea cum rectis AD, BD conſtituat angulos DQP, DMP æquales, ad MQ demitto normales AR, BN, DO. & adhibeo DO pro

ordinatim applicata, ac PO pro indefinita linea cui inſiſtit.

Sit PO = x	Ob ſim. tr. BNM,
DO = y	DOM, erit BN : DO
AR vel BN = b	= MN : MO. & *divi-*
PR vel PN = c	*dendo* DO-BN (= $y-b$:

DO (= y) = MO-MN (= $c-x$) : MO. Quare MO = ($cy-xy$) : ($y-b$). Dein ex altera parte, ob ſim. tr. ARQ, DOQ, erit AR : DO = RQ : QO, & *componendo* DO + AR (= $y+b$) : DO (= y) = QO + QR (= OR ſeu $c+x$) : QO. Quare QO = ($cy+xy$) : ($y+b$). Denique, ob æquales ang. DMQ, DQM, æquantur MO & QO hoc eſt ($cy-xy$) : ($y-b$) = ($cy+xy$) : ($y+b$). Ergo $xy = bc$ notiſſima æquatio ad Hyperbolam.

Quin & locus puncti D ſine Algebra prodire potuit. Nam ex ſuperioribus DO-BN : ON = DO : MO (= QO) = DO + AR : OR. hoc eſt DO-BN : DO + BN = ON : OR. & mixtim DO : NB = $\frac{1}{2}$ON + $\frac{1}{2}$OR (= NP) : $\frac{1}{2}$OR - $\frac{1}{2}$ON (= OP). Adeoque DO x OP = BN x NP.

PROB.

PROB. XLII.

Locum verticis trianguli invenire cujus baſis datur & angulorum ad baſem unus dato angulo differt a duplo alterius?

TAB. III. FIG. 9.

Sit ABD triang. iſtud, AB baſis biſecta in P, APQ vel BPM triens anguli dati, quo angulus DBA excedit duplum anguli DAB; & angulus DMQ erit duplus anguli DQM. Ad MQ demitte perp. AR, BN, DO; & ang. DMQ biſeca recta MS occurrente DO in S; & erunt tr. DOQ, SOM ſimilia. Hinc OQ : OM = OD : OS & *dividendo* OQ−OM : OM = SD : OS = (per 3. VI. Elem.) DM : OM. Quare (per 9. V. Elem.) OQ − OM = DM. jam

Dic PO $= \sqrt{x}$
OD $= \sqrt{y}$
AR vel BN $= \sqrt{b}$
PR vel PN $= \sqrt{c}$

Erit ut in prob. ſup. OM $= (\sqrt{cy} - \sqrt{xy}) : (\sqrt{y} - \sqrt{b})$. Et OQ $= (\sqrt{cy} + \sqrt{xy}) : (\sqrt{y} + \sqrt{b})$. Adeoque OQ − OM $= (-2\sqrt{bcy} + 2y\sqrt{x}) : (y - b)$. Pone jam DO$q$ + DMq = DM, hoc eſt $y + \frac{c - 2\sqrt{cx} + x}{y - 2\sqrt{by} + b} y = \frac{4bc - 8\sqrt{bcxy} + 4xy}{yy - 2by + bb} y.$

ſeu 1

ſeu $1 + \frac{\sqrt{c} - \sqrt{x}}{\sqrt{y} - \sqrt{b}} \times \frac{\sqrt{c} - \sqrt{x}}{\sqrt{y} - \sqrt{b}} = \frac{4bc - 8\sqrt{bcxy} + 4xy}{(y - b) \times (y - b)}$ adeoque (per 25.) $y - 2\sqrt{by} + b + c - 2\sqrt{cx} + x = \frac{4bc - 8\sqrt{bcxy} + 4xy}{(\sqrt{y} + \sqrt{b}) \times (\sqrt{y} + \sqrt{b})}$. Et $y^2 * + (-3x - 2b - 2\sqrt{cx})y + (2c\sqrt{b} + 2x\sqrt{b} + 4\sqrt{bcx})\sqrt{y} + b^2 + bx - 2b\sqrt{cx} - 3bc = 0$. Divide omnia per $\sqrt{y} - \sqrt{b}$ & evadet $\sqrt{y^3} + y\sqrt{b} + (c - b - 3x - 2\sqrt{cx})\sqrt{y} + (3c\sqrt{b} - \sqrt{b^3} + 2\sqrt{bcy} + x\sqrt{b}) = 0$. Quare punctum D eſt ad Curvam trium dimenſionum, quæ tamen evadit Hyperbola ubi ang. BPM ſtatuitur nullus, ſive angulorum ad baſem unus DBA duplus alterius DAB. Tunc enim BN ſeu $\sqrt{b}$ evaneſcente, fiet æquatio $y = 3x + 2\sqrt{cx} - c$.

TAB. III. FIG. 10. Ex hujus autem æquationis conſtructione tale elicitur Theorema. Si centro C, Aſymptotis CS, CT angulum SCT 120 graduum continentibus deſcribatur Hyperbola quævis DV cujus ſemiaxes ſint CV, CA: produc CV ad B ut ſit VB = VC, & ab A & B actis utcumque rectis AD, BD concurrentibus ad Hyperbolam, erit angulus BAD (= ½ ang. ABD) = ⅓ anguli ADE quem recta AD comprehendit cum

BD

BD producta. Quod intellige de Hyperbola quæ tranſit per punctum V. Nam ſi ab iisdem punctis A & B actæ rectæ Ad, Bd conveniant ad conjugatam Hyperbolam quæ tranſit per A: tunc externorum angulorum trianguli ad baſim, ille ad B erit duplus alterius ad A.

PROB. XLIII.

Circulum per data duo puncta deſcribere, qui rectam poſitione datam continget? TAB. IV. FIG. 1.

Sunto A & B puncta data, & EF recta poſitione data & requiratur circulum ABE per iſta puncta deſcribere qui contingat rectam iſtam FE. junge AB & eam biſeca in D. ad D erige normalem DF occurrentem rectæ FE in F, & circuli centrum incidet in hanc DF, puta in C. junge ergo CB & ad FE demitte CB normalem eritque E punctum contactus ac CB = CE, utpote radii circuli quæſiti. Jam datis punctis A, B, D & F, ut determinetur centrum circuli, quæratur DC.

Sit DB $= \sqrt{a}$ DF $= \sqrt{b}$ DC $= \sqrt{x}$	In tr. CDB, ob ang ad D rectum eſt $\sqrt{a+x} =$ CB & $\sqrt{b} - \sqrt{x} =$ DF−DC $=$ CF.

Et

Et in tr. rect. CFE, ob datos ang. datur ratio laterum CF & CE quam pone $\sqrt{d}$ ad $\sqrt{e}$, & erit $CE = (\sqrt{eb} - \sqrt{ex}) : \sqrt{d} = CB = \sqrt{a+x}$. Ergo $eb - 2e\sqrt{bx} + ex = ad + dx$. Et $\sqrt{x} = (-e\sqrt{b} + \sqrt{ebd + aed - ad^2}) : (d-e)$. Inventa DC datur centrum C quo circulus per puncta A & B deſcribendus eſt ut contingat rectam FE.

PROB. XLIV.

TAB. IV. FIG. 2. *Circulum per datum punctum deſcribere qui rectas duas poſitione datas contingat?*

Hoc Problema reſolvitur ut præcedens. Nam dato puncto A datur & aliud punctum B.

PROB. XLV.

TAB. IV. FIG. 3. *Circulum per data duo puncta deſcribere qui alium circulum poſitione datum continget?*

Sint A & B puncta data, EK circulus poſitione & magnitudine datus, F centrum ejus, ABE circulus quæſitus per puncta A & B tranſiens ac tangens alterum circulum in E, & C centrum ejus. Ad AB productam demitte perpend. CD & FG & age CF ſecantem circulos in puncto contactus E, ac age etiam FH pa-

parallelam DG & occurrentem CD in H. His conſtructis.

Dic AD vel DB $= \sqrt{a}$	Erit CH ($=$ CD $-$ FG) $= \sqrt{x} - \sqrt{c}$. &
BG, vel HF $= \sqrt{b}$	CFq ($=$ CHq + HFq)
GF $= \sqrt{c}$	$= x - 2\sqrt{cx} + c + b$,
radium dat. EF $= \sqrt{d}$	atque CBq ($=$ CDq
DC $= \sqrt{x}$	

+ DBq) $= x + a$, & CB ($=$ CE) $= \sqrt{x+a}$. Sed CE + EF $=$ CF $= \sqrt{x+a} + \sqrt{d} = \sqrt{(x - 2\sqrt{cx} + c + b)}$. Ergo $x + a + d + 2\sqrt{dx + ad} = x - 2\sqrt{cx} + c + b$. Eſto $c + b - a - d = 2g$, eritque $2\sqrt{dx + ad} = -2\sqrt{cx} + 2g$. Et $dx + ad = cx + g^2 - 2\sqrt{cgx}$, Et $\sqrt{x} = (-g\sqrt{c} + \sqrt{ggd + acd - add}) : (d - c)$.

Inventa igitur $\sqrt{x} =$ DC, biſeca AB in D & ad D erige perp. DC. Dein Centro C per punctum A vel B deſcribe circulum ABE, nam hic continget alterum circulum EK & tranſibit per utrumque punctum A. & B. Q. E. F.

PROB. XLVI.

Circulum per datum punctum deſcribere qui datum circulum & rectam lineam poſitione datam continget. TAB. IV. FIG. 4.

Sit

Sit circulus iſte deſcribendus BC, ejus centrum C, punctum per quod deſcribi debet B, recta quam contingit AD, punctum contactus D, circulus quem continget GEM, ejus centrum F & punctum contactus E, junge CB, CD, CF & CD erit perpend. ad AD, atque CF ſecabit circulos in puncto contactus E. Produc CD ad Q ut ſit DQ = EF & per Q age QN parallelam AD. Denique à B & F ad AD & QN demitte perpendicula BA, FN & à C ad AB & FN perpendicula CK, CL. Et cum ſit BC = CD vel AK, erit BK (= AB − AK) = AB − BC, adeoque BK*q* = AB*q* − 2AB × BC + BC*q*. Aufer hoc de BC*q* & reſtabit 2AB × BC − AB*q* pro quadrato de CK. Eſt itaque AB × (2BC − AB) = CK*q*; & eodem argumento erit FN × (2FC − FN) = CL*q*, atque adeo CK*q* : AB, + AB, = 2BC & CL*q* : FN, + FN = 2FC. Quare ſit

$AB = \sqrt{a}$
$CK = \sqrt{y}$
$FN = \sqrt{b}$
$KL = \sqrt{c}$
$LC = \sqrt{c} - \sqrt{y}$

Erit $y : 2\sqrt{a} + \frac{1}{2}\sqrt{a} =$ BC. & $(a - 2\sqrt{cy} + y) : 2\sqrt{b} + \frac{1}{2}\sqrt{b} =$ FC. & FC − BC $= (a - 2\sqrt{cy} + y + b) : 2\sqrt{b} - (y - a) : 2\sqrt{a} =$ EF. Jam ſi puncta ubi FN producta ſecat rectam AD & circulum GEM notentur litteris H, G & M & in HG pro-

producta capiatur HR = AB, cum ſit HN = (DQ = EF) = GF addendo utrimque FH erit FN = GH, adeoque AB − FN = (HR − GH) = GR & AB − FN + 2EF = RM, hoc eſt $\sqrt{a} - \sqrt{b} + 2EF = RM$ & $\frac{1}{2}\sqrt{a} - \frac{1}{2}\sqrt{b} + EF = \frac{1}{2}RM$ quod ergo dic $\frac{1}{2}\sqrt{d}$ & ob inventam EF ſupra, erit $\sqrt{abd} = c\sqrt{a} - 2\sqrt{acy} + y\sqrt{b}$. Et $y\sqrt{a} - y\sqrt{b} = 2\sqrt{acy} - c\sqrt{a} + \sqrt{abd}$ & $\sqrt{y} = (\sqrt{ac} \pm \sqrt{(a\sqrt{bd} - b\sqrt{ad} + c\sqrt{ab})}) : (\sqrt{a} - \sqrt{b})$ = CK vel AD. Cape AD = $\sqrt{y}$, ad D erige perpend. DC (= BC) = $\frac{1}{2}$AB + KCq : 2AB & centro C, intervallo CB vel CD deſcribe circulum BDE, nam hic tranſiens per datum punctum B tanget rectam AD in D & circulum GEM in E. Q. E. F.

Hinc circulus etiam deſcribi poteſt qui duos datos circulos & rectam poſitione datam continget. Sint enim circuli dati RT, SV, eorum centra B & F, & recta poſitione data PQ. Centro F, radio FS − BR deſcribe circulum EM. A puncto B ad rectam PQ demitte perpend. BP & producto eo ad A ut ſit PA = BR per A age AH parallelam PQ & circulus deſcribatur qui tranſeat per punctum B, tangatque rectam AH & circulum EM. Sit ejus centrum C; junge BC ſecantem circulum RT in R & eodem centro C, radio TAB. IV. FIG. 5.

dio vero CR deſcriptus circulus RS tanget & circulos RT, SV, & rectam PQ, ut ex conſtructione manifeſtum eſt.

PROBLEMA XLVII.

TAB. IV. FIG. 6. *Circulum deſcribere qui per datum punctum tranſibit & alios duos poſitione & magnitudine datos circulos continget?*

Eſto punctum datum A, ſintque circuli poſit. & magnit. dati TIV & RHS, centra eorum C & B, circulus deſcribendus AIH, centrum ejus D & puncta contactus I & H: junge AB, AC, AD, DB, ſecetque AB producta circulum RHS in punctis R & S, & AC producta circulum TIV in T & V. Et a punctis D & C. demiſſis perpend. DE ad AB & DF ad AC occurrente AB in G atque CK ad AB; in tr. ADB erit $ADq - DBq + ABq = 2AE \times AB$, per 13. II. Elem. ſed $DB = AD + BR$, adeoque $DBq = ADq + 2AD \times BR + BRq$. Aufer hoc de $ADq + ABq$, & reſtabit $ABq - 2AD \times BR, - BRq$ pro $2AE \times AB$. Eſt & $ABq - BRq = (AB + BR) \times (AB - BR) = AR \times AS$. Quare $AR \times AS, - 2AD \times BR = 2AE \times AB$. Et $(AR \times AS, - 2AB \times AE) : BR = 2AD$.

Et ſimili ratiocinio in triang. ADC emer-

emerget rurſus 2AD = (TAV - 2CAF) : CT. Quare (RAS - 2BAE) : BR = (TAV - 2CAF) : CT. Et TAV : CT, - RAS : BR, + 2BAE : BR = 2(AF : CT). Et (TAV : CT, - RAS : BR, + 2BAE : BR) x CT : 2AC = AF. Unde cum ſit AK : AC = AF : AG. erit AG = (TAV : CT, - RAS : BR, + 2BAE : BR) x CT : 2AK. Aufer hoc de AE ſeu 2KAE : CT, x CT : 2AK & reſtabit GE = (RAS : BR, - TAV : CT, - 2BAE : BR, + 2KAE : CT) x CT : 2AK. Unde cum ſit KC : AK = GE : DE; erit DE = (RAS : BR, - TAV : CT, - 2BAE : BR, + 2KAE : CT) x CT : 2KC. In AB cape AP quæ ſit ad AB ut CT ad BR & erit 2PAE : CT = 2BAE : BR, adeoque 2PK x AE : CT = 2BAE : BR, - 2KAE : CT. adeoque DE = (RAS : BR, - TAV : CT, - 2PK) x AE : CT) x CT : 2KC. Ad AB ergo erige perpend. AQ = (AS : BR, - TAV : CT) x CT : 2KC & in eo cape QO = PK x AE : KC & erit AO = DE.

Junge DO, DQ, CP, & tr. DOQ, CKD erunt ſimilia, quippe quorum anguli ad O & K ſunt recti & latera PK = AE vel DO : QO proportionalia. Ergo angul. OQD = ang. KPC & proinde QD perp. eſt ad CP. Hinc ſi agatur AN parall. CP & occurrens QD in N, angulus ANQ erit rectus & tr.

 AQN,

AQN, PCQ ſimilia. Adeoque PC : KC = AQ : AN. Unde cum AQ ſit (RAS : BR, -TAV : CT) x CT : 2KC, AN erit (RAS : BR, -TAV : CT) x CT : 2PC. Produc AN ad M ut ſit NM = AN & erit AD = DM, adeoque circulus quæſitus tranſibit per punctum M. Cum ergo punctum M datum ſit, ex hjs ſine ulteriori Analiſi talis emerget problematis reſolutio.

In AB cape AP, quæ ſit ad AB ut CT ad Br; junge CP eique parallelam age AM quæ ſit ad RAS : BR, -TAV : CT, ut CT ad PC & ope Prob. 45 per puncta A & M deſcribe circulum AIHM qui tangat alterum circulorum TIV, RHS & idem circulus tanget utrumque. Q. E. F.

Et hinc circulus etiam deſcribi poteſt qui tres circulos poſitione & magnitudine datos continget. Sunto trium datorum circulorum radii A, B, C & centra D, E, F. Centris E & F, radiis B ± A, C ± A deſcribantur duo circuli & tertius circ. qui hoſce tangat tranſeatque per punctum D. Sit hujus radius G & centrum H & centro eodem H radio G ± A deſcriptus circulus continget tres primos circulos. *Ut opport.*

PROB.

PROB. XLVIII.

Si ad extremitates fili D AE *circa paxillum* A *labentis appendantur pondera duo* D *&* E, *quorum pondus* E *labitur per lineam obliquam* BG: *invenire locum ponderis* E, *ubi pondera hæc in æquilibrio conſiſtunt.* TAB. IV. FIG. 7.

Puta factum, & ipſi AD age parallelam EF quæ ſit ad AE ut pondus E ad pondus D, & a punctis A & F ad lineam BG demitte perp. AB, FG. jam cum pondera ex hipoth. ſint ut lineæ AE, EF hæc per has lineas exponantur, ſcilicet pondus D per AE & pondus E per EF, Ergo corpus E proprii ponderis vi directa EF tendit verſus F & vi obliqua EG tendit verſus G, & idem corpus E, ponderis D vi directa AE trahitur verſus A & vi obliqua BE trahitur verſus B. Cum itaque pondera ſe mutuo ſuſtineant in æquilibrio, vis qua pondus E trahitur verſus B æquatur vi contrariæ qua tendit verſus G, hoc eſt BE = EG. Jam vero datur ratio AE ad EF ex hypotheſi., & ob datum ang. FEG datur etiam ratio FE ad EG = BE. Ergo datur ratio AE ad BE. Datur etiam AB longitudine, & inde tr. ABE & punctum E facile dabitur.

$AB = \sqrt{a}$) Erit $AE = \sqrt{a+x}$. Sit inſuper
$BE = \sqrt{x}$) $AE : BE = \sqrt{d} : \sqrt{e}$, eritque
$\sqrt{ae+ex} = \sqrt{dx}$ & $ae+ex = dx$ & $\sqrt{x} = \sqrt{ae : (d-e)}$. Inventa eſt igitur longitudo BE quæ determinat locum ponderis E. Q. E. F.

TAB.IV. FIG. 8. Quod ſi pondus utrumque per lineam obliquam deſcendat computum ſic inſtitue. Sint CD, BE obliquæ lineæ poſitione datæ per quas pondera iſta D & E deſcendunt. A paxillo A ad has lin. demitte perp. AC, AB, iisque productis occurrant in punctis G & H lineæ EG, DH a ponderibus perpendiculariter ad Horizontem erectæ, & vis qua pondus E conatur deſcendere juxta lineam perpend. hoc eſt tota gravitas ipſius E, erit ad vim qua pondus idem conatur deſcendere juxta lin. obliquam BE, ut GE ad BE, atque vis qua conatur juxta eandem obliq. BE deſcendere erit ad vim qua conatur juxta lineam AE deſcendere, hoc eſt, ad vim qua filium AE diſtenditur, ut BE ad AE. Adeoque gravitas ipſius E erit ad tenſionem fili AE ut GE ad AE. Et ead. ratione gravitas ipſius D erit ad tenſionem fili AD ut HD ad AD. Sit itaque fili totius longitudo $DA + AE = \sqrt{c}$, & pars $AE = \sqrt{x}$.

DA

$DA + AE = \sqrt{c}$	$AD = \sqrt{c} - \sqrt{x}$ & ob
$AB = \sqrt{a}$	$AEq - ABq = BEq$ &
$AC = \sqrt{b}$	$ADq - ACq = CDq$,
$AE = \sqrt{x}$	erit $BE = \sqrt{x - a}$

& $CD = \sqrt{c - 2\sqrt{cx} + x - b}$. Adhæc cum triang. BEG & CDH dentur ſpecie, ſit $BE : EG = \sqrt{f} : E$ & $CD : DH = \sqrt{f} : \sqrt{g}$, & erit $EG = E\sqrt{x - a} : \sqrt{f}$ & $DH = \sqrt{x - 2\sqrt{cx} + c - b} \times \sqrt{g : f}$. Quare cum ſit GE ad AE ut pondus E ad tenſionem AE, & HD: AD ut pondus D ad tenſionem AD & tenſiones iſtæ æquentur inter ſe, erit $E\sqrt{x} : \sqrt{(x - a)} \times E : \sqrt{f} =$ tenſioni AE $=$ tenſioni AD $= (D\sqrt{c} - D\sqrt{x}) : \sqrt{(x - 2\sqrt{cx}\ c - b)} \times \sqrt{g : f}$: Hinc $D \times (\sqrt{c} - \sqrt{x}) \times \sqrt{x - a} = \sqrt{gx} \times \sqrt{x - 2\sqrt{cx} + c - b}$. ſeu $D \times \sqrt{c - 2\sqrt{cx} + x} \times \sqrt{x - a} = \sqrt{gx} \times \sqrt{x - 2\sqrt{cx} + c - b}$. Ergo $D^2 \times (cx - 2x\sqrt{cx} + 2a\sqrt{cx} + x^2 - ax - ac) = gx^2 - 2gx\sqrt{cx} + gcx - gbx$.

$$\text{Et..}\quad \begin{matrix} gx^2 & -2gx\sqrt{cx} & \begin{matrix} +gcx \\ -gbx \\ -D^2cx \\ +D^2ax \end{matrix} & -2D^2a\sqrt{cx} \\ -D^2x^2 & +2D^2x\sqrt{cx} & & +D^2ac = 0. \end{matrix}$$

Vis caſum quo hoc probl. per Regulam & Circinum conſtrui queat? pone pondus D ad pondus E ut (BE: EG) ad

(CD: DH) & evadet $\sqrt{g} = D$ in æquatione ſuperiori. adeoque $\sqrt{x} = \sqrt{ac} : (\sqrt{a} + \sqrt{b})$.

PROB. XLIX.

TAB. IV. FIG. 9. *Si ad filum* DACBF *circa paxillos duos* A, B *labile appendantur tria pondera* D, E, F; D *&* F *ad extremitates fili &* E *ad medium ejus punctum* C, *inter paxillos poſitum; ex datis ponderibus & ſitu paxillorum invenire ſitum puncti* C *ad quod medium pondus appenditur ubi pondera conſiſtunt in æquilibrio?*

Cum tenſio fili AC æquatur tenſioni fili AD & tenſio fili BC tenſioni fili BF, tenſiones filorum AC, BC, EC erunt ut pondera D, F, E. In eadem ponderum ratione cape partes filorum CG, CH, CI. Comple tr. GHI. Produc IC donec ea occurrat GH in K & erit GK = KH & CK = ½ CI, adeoque C centrum Gravitatis trianguli GHI. Nam per C agatur ipſi CE perpendiculare PQ & huic a punctis G & H perpendicularia GP, HQ, & ſi vis qua filum AC vi ponderis D trahit punctum C verſus A, exponatur per lineam GC, vis qua filum iſtud trahet idem punctum verſus P exponetur per lineam CP & vis qua trahit illud verſus K exponetur

ponetur per lineam GD. Et ſimiliter vires quibus filum BC vi ponderis F trahit idem pondus C verſus P, Q & K exponantur per lineas CH, CQ, HQ; & vis qua filum CE vi ponderis E trahit punctum illud C verſus E exponatur per lin. CI. jam cum punctum C viribus æquipollentibus ſuſtineatur in æquilibrio, ſumma virium quibus fili AC & BC ſimul trahunt punctum C verſus K æquabitur vi contrariæ qua filum EC trahit illud verſus E, hoc eſt, ſumma $GP + HQ = CI$; & vis qua filum AC trahit punctum C verſus P æquabitur vi contrariæ qua filum BC trahit idem punctum C verſus Q, hoc eſt, linea $PC = CQ$.

Quare cum PG, CK & QH parallelæ ſint, erit etiam $GK = KH$ & $CK = \frac{1}{2}GP + \frac{1}{2}HQ = \frac{1}{2}Cl$. Q. E. D. Reſtat ergo tr. GCH determinandum, cujus latera GC & HC dantur una cum linea CK quæ a vertice C ad medium baſis ducitur. Demittatur itaque a vertice C ad baſim GH perp. CL. & erit $(GCq - CHq) : 2GH = KL = (GCq - KCq - GKq) : 2GK$. Sed $2GK = GH$: Ergo (Ax.). $GCq - 2KCq + CHq = 2GKq$, ſive $\sqrt{\frac{1}{2}GCq - KCq + \frac{1}{2}CHq} = GK$. Invento GK vel KH dantur ſimul anguli GCK, KCH ſive DAC, FBC. Quare a punctis A & B in datis iſtis angulis DAC,

FBC duc lineas AC, BC concurrentes in puncto C & iſtud C erit punctum quod quæritur.

Cæterum quæſtiones omnes quæ ſunt ejusdem generis non ſemper opus eſt per Algebram ſigillatim ſolvere: ſed ex ſolutione unius plerumque conſectatur ſolutio alterius.

TAB. IV. FIG. 10. EXEMPLUM. *Filo* ACDB *in datas partes* AC, CD, DB *diviſo & extremitatibus ejus ad paxillos duos* A, B *poſitione datos ligatis; ſi ad puncta diviſionum* C *ac* D *appendantur pondera duo* E *&* F: *ex dato pondere* F *& ſitu punctorum* C *ac* D, *cognoſcere pondus* E.?

Solutio hujus Quæſtionis ſat facile colligitur ex præcedenti. Produc lineas AC, BD, donec occurrant lineis DF, CE in G & H, & erit pondus E ad pondus F ut DG ad CH. Hinc obiter patet ratio componendi ſtateram ex ſolis filis, qua pondus corporis cujusvis E, ex unico dato pondere F cognoſci poteſt.

PROB. L.

Lapide in Puteum decidente, ex ſono lapidis fundum percutientis, altitudinem putei cognoſcere.

Sit

Sit x altitudo putei & ſi lapis motu uniformiter accelerato deſcendat per ſpatium quodlibet datum a in tempore dato b & ſonus motu uniformi percurrat idem ſpatium a in tempore dato d, erit $\sqrt{a} : b = \sqrt{x} : b\sqrt{x} : a$. Nam tempora deſcenſus gravium ſunt ut radices ſpatiorum. Erit etiam $a : d = x : (dx : a)$. Nam tempora aſcenſus ſoni ſunt ut ſpatia. Sed $dx : a, + b\sqrt{x : a} = t$, tempori totali aſcenſus & aſcenſus, quod horologio cognoſci poteſt. Ergo $x + b\sqrt{ax} - adt = 0$, & $\sqrt{x} = (-b\sqrt{a} + \sqrt{abb + 4adt}) : 2d$, cujus quadratum eſt altitudo quæſita.

PROB. LI.

Dato globo A, *poſitione parietis* DE *& centri globi* B *à pariete diſtantia* BD: *invenire molem globi* B, *ea lege ut in ſpatiis liberis & vi gravitatis deſtitutis, ſi globus* A *cujus centrum in linea* BD *quæ ad parietem perpendicularis eſt, ultra* B *producta conſiſtit, uniformi cum motu verſus* D *feratur donec is impingat in alterum quieſcentem globum* B; *globus iſte* B *poſtquam reflectitur a pariete, denuo occurrat globo* A *in dato puncto* C. TAB. V. FIG. 1.

Sit

Sit globi A celeritas ante reflexionem a, & erit per prob. XII. pag. 67. celeritas globi A poſt reflexionem $(aA - aB)$: $(A + B)$ & celeritas globi B poſt reflexionem $2aA$: $(A + B$. Ergo cel. globi A ad cel. globi B eſt ut $A - B$ ad $2A$. In GD cape $gD = GH$ diametro nempe globi B & celeritates iſtæ erunt ut GC ad $Gg + gC$. Nam ubi globus A impegit in globum B, punctum G quod in ſuperficie globi B exiſtens movetur in linea AD, perget per ſpatium gG antequam globus ille B impinget in parietem & per ſpatium gC poſtquam a pariete reflectitur; hoc eſt, per totum ſpatium $Gg + gC$ in eodem tempore quo globi A punctum F perget per ſpatium GC, eo ut globus uterque rurſus conveniant & in ſe mutuo impingant in puncto dato C. Quare cum dentur intervalla BC & CD,

Dic. $BC = m$	Erit $GC = m + x$ &
$BD + CD = n$	$Gg + gC = GD + DC$
$BG = x$	$-2gD = GB + DC$

$-2GH = x + n - 4x = n - 3x$. Supra erat $A - B$ ad $2A$ ut celer. gl. A ad celer. gl. B, & cel. gl. A ad cel. gl. B ut GC ad $Gg + gC$. Ergo cum ſit $GC = m + x$ & $Gg + gC = n - 3x$, erit $A - B$ ad $2A$

ut

ut $m+x$ ad $n-3x$. Porro gl. A eſt ad gl. B ut cubus radii ejus AF ad cubum radii alterius GB, hoc eſt ut s^3 ad x^3 ſi ponas AF $= s$. Ergo $(s^3-x^3):2s^3$ ($=$ A$-$B: 2A) $= m+x : n-3x$ & per reductionem $3x^4-nx^3-5s^3x+s^3n-2s^3m=0$. Cujus æquationis conſtructione dabitur globi B ſemidiameter x, adeoque globus ille. Q. E. F.

Nota vero 1° quod ſi punctum C jacet ad contrarias partes globi B debet ſignum quantitatis m mutari, ut fiat $+2s^3m$ pro $-2s^3m$ in æquatione noviſſima. Quod 2° ſi dato globo B quæreretur gl. A, ea lege ut globi duo poſt reflexionem convenirent, in eadem, ſupponendo x dari & s quæri, emergeret $(3x^4-nx^3):(5x-n+2m)=s^3$; ubi per ſolam extractionem radicis cubicæ obtinebitur s. Quod 3° ſi datis globis, quæras punctum C in quo poſt reflexionem ambo in ſe mutuo impingent, (ob ſupra A$-$B: 2A $=$ GC: G$g+g$C) erit invertendo & componendo 3A$-$B: A$-$B $=$ 2Gg: GC *diſtantiam quæſitam.*

PROB. LII.

Si duo globi A *&* B *tenui jungantur filo* PQ *& pendente globo* B *à globo* A, *ſi dimittatur globus* A, *ita ut globus uterque* TAB. V. FIG. 2.

ſimul

ſimul ſola gravitatis vi in ead linea perp. PQ cadere incipiat; dein globus inferior B, poſtquam a fundo ſeu plano horizontali FG rurſum reflectitur, ſuperiori decidenti globo A occurrat in puncto quodam D: ex data fili longitudine PQ & puncti illius D diſtantia DF, invenire altitudinem PF, a qua globus ſuperior A ad hunc effectum demitti debet?

Sit fili PQ longitudo a. In perpendiculo PQRF ad F ſurſum cape FE = QR globi inferioris diametro, ita ut cum globi illius punctum infimum R incidit in fundum ad F, punctum ejus ſupremum Q occupet locum E, ſitque ED diſtantia per quam globus ille poſtquam a fundo reflectitur aſcendendo tranſit antequam globo ſuperiori decidenti occurrat in puncto D. igitur ob datam puncti D à fundo diſtantiam DF globique inferioris diametrum EF, dabitur eorum differentia DE. Sit ea = b, ſitque altitudo per quam globus ille inferior antequam impingit in fundum cadando deſcribit RF vel QE = x, ſiquidem ea ignoretur. Et invento x erit altitudo x + EF + PQ = PF à qua globus ſuperior ad effectum deſideratum demitti debet. Hinc eſto.

$PQ = a$, $DE = b$, $QE = x$.

Erit $PE = a + x$ & $PE - DE = PD = a + x - b$. Sed tempus deſcenſus globi A eſt ut radix ſpatii cadendo deſcripti ſeu $\sqrt{a + x - b}$ & tempus deſcenſus globi B ut radix ſpatii cadendo deſcripti ſeu $\sqrt{x}$, & tempus aſcenſus ejusdem ut differentia radicis illius & radicis ſpatii quòd cadendo tantum à Q ad D deſcriberetur. Nam hæc differentia eſt ut tempus deſcenſus à D ad E quod æquatur tempori aſcenſus ab E ad D. Eſt autem differentia illa $\sqrt{x} - \sqrt{x - b}$. Unde tempus deſcenſus & aſcenſus conjunctim erit ut $2\sqrt{x} - \sqrt{x - b}$. Quare cum hoc tempus æquetur tempori deſcenſus globi ſuperioris erit $\sqrt{a + x - b} = 2\sqrt{x} - \sqrt{x - b}$. & $a + x - b = 5x - b - 4\sqrt{xx - bx}$ & (Axiom. II. III.) $x = a^2 : (8a - 16b)$ ſeu QE. Q.E.I.

Quod ſi ex dato $QE = x$ quæreretur fili longitudo $PQ = a$ cum ſit $a^2 = 8ax - 16bx$ erit (15.) $a = 4x - 4\sqrt{xx - bx} = PQ$. Si vero ex datis x & a quæras punctum D in quo globus ſuperior in inferiorem incidit, puncti illius à dato puncto E diſtantia DE ſeu b ex præcedenti æquatione $a^2 = 8ax - 16bx$ eruetur

(Ax.

(Ax. III.) nam $b = (8ax - a^2) : 16$. Ergo $16x : 8x - a = a : b =$ DE. Q. E. I.

Hactenus ſuppoſui globos tenui filo connexos ſimul dimitti. Quod ſi nullo connexi filo diverſis temporibus dimittantur, ita ut globus ſuperior A ex. gr. prius dimiſſus deſcenderit per ſpatium PT antequam globus alter incipiat cadere, & ex datis diſtantiis PT, PQ ac DE quæratur altitudo PF, a qua globus ſuperior dimitti debet ea lege ut in inferiorem incidat ad punctum D...

Sit QP $= a^2$ DE $= b^2$ PT $= c^2$ QE $= x^2$	Erit PD $= a^2 + x^2 - b^2$ ut ſupra. Et tempora quibus globus ſuperior cadendo deſcribat ſpatia PT ac TD &

globus inferior prius cadendo, dein reaſcendendo deſcribat ſummam ſpatiorum QE+ED, erunt ut $\sqrt{PT}$, $\sqrt{PD} - \sqrt{PT}$, & $2\sqrt{QE} - \sqrt{QD}$, hoc eſt ut c, $\sqrt{a^2 + DE - b^2} - c$ & $2x - \sqrt{x^2 - b^2}$. At illa duo tempora (ob ſpatia TD, & QE+ED ſimul deſcripta) ſunt æqualia. Ergo $\sqrt{a^2 + x^2 - b^2} - c = 2x - \sqrt{x^2 - b^2}$. & $2x + c = \sqrt{a^2 + x^2 - b^2} + \sqrt{x^2 - b^2}$ & $4x^2 + 4cx + c^2 = a^2 + 2x^2 - 2b^2 + 2\sqrt{a^2x^2 + x^4 - 2b^2x^2 - a^2b^2 + b^4}$. Et $2x^2 + 4cx + c^2 - a^2 + 2b^2 =$

$2\sqrt{}$

$2\sqrt{a^2x^2 + x^4 - 2b^2x^2 + b^4}$. pro datis $c^2 - a^2 + 2b^2$ ſcribo $2d$ fitque $x^2 + 2cx + d^2 = \sqrt{a^2x^2 + x^4 - 2b^2x^2 + b^4}$. & $x^4 + 4c^2x^2 + d^4 + 4cx^3 + 2d^2x^2 + 4cd^2x = a^2x^2 + x^4 - 2b^2x^2 + b^4$. Quare $4cx^3 + (4d^2 + 3c^2)x^2 + 4cd^2x + d^4 - b^4 = 0$. Cujus æquationis conſtructione dabitur x ſeu QE cui adde datas diſtantias PQ & EF & habebitur altitudo PF. Q. E. I.

PROB. LIII.

Si globi duo quieſcentes ſuperior A, *& infe-*TAB. V. FIG. 1. *rior* B *diverſis temporibus demittantur & globus infer. eo temporis momento cadere incipiat ubi ſuperior cadendo deſcripſit ſpatium* PT, *invenire loca* α, β *quæ globi illi cadentes occupabunt ubi eorem intervallum* $\pi\chi$ *dato æquale eſt?*

PT $= a$ PQ $= b$ $\pi\chi = c$ P$\pi = x$	Cum dentur differentiæ PT, PQ, & $\Pi\chi$ dic primam a, 2dam b, 3am c, & pro PΠ ſeu ſpatio quod globus ſup. an-

tequam pervenit ad locum quæſitum α cadendo deſcribit ponatur x. Jam tempora quibus globus ſup. deſcribit ſpatia PT, PΠ, TΠ, & inferior ſpatium Qχ ſunt ut $\sqrt{PT}$, $\sqrt{P\Pi}$, $\sqrt{P\Pi} - \sqrt{PT}$, & $\sqrt{Q\chi}$. Quorum temporum poſteriora duo, eo quod

globi cadendo ſimul deſcribant ſpatia $T\pi$ & $Q\chi$ ſunt æqualia. Unde & $\sqrt{P\Pi} - \sqrt{PT} = \sqrt{Q\chi}$. Erat $P\pi = x$ & $PT = a$ & $P\pi + c - b = x + c - b = Q\chi$. Ergo $\sqrt{x} - \sqrt{a} = \sqrt{x + c - b}$ & $- 2\sqrt{ax} + a = c - b$. & $\sqrt{x} = (a + b - c) : 2\sqrt{a}$ & $4a : (a + b - c) = (a + b - c) : x$. Ex invento autem $x = P\Pi$ datur globi ſup. decidentis locus quæſitus α. Et per locorum diſtantiam ſimul datur locus inferioris β.

Et hinc ſi quæratur locus ubi globus ſup. cadendo tandem impinget in infer. ponendo diſtantiam $\Pi\chi$ nullam eſſe ſeu delendo c dic $4a : (a + b) = (a + b) : x$ & punctum x erit quod quæris.

Et viciſſim ſi detur punctum illud Π vel χ in quo globus ſup. incidit in infer. & quæratur locus T quem globi ſup. decidentis punctum imum P tunc occupabat cum globus infer. incipiebat cadere, ob $4a : a + b = a + b : x$, erit $a^2 = 4ax - 2ab - b^2$ & $a = 2x - b - 2\sqrt{x^2 - bx}$. Cape ergo $V\Pi$ mediam prop. inter $P\Pi$ & $Q\Pi$ & verſus V cape $VT = VQ$ & erit T punctum quod quæris. Nam erit $V\Pi = \sqrt{P\pi \times Q\pi} = \sqrt{x} \times \sqrt{x - b}$ ſeu $\sqrt{xx - bx}$, cujus duplum ſubductum de $2x - b$ ſeu de $2P\pi - PQ$ id eſt de $PQ + 2Q\pi$ relinquit $PQ - 2VQ$ ſeu $PV - VQ = PT$.

Si

Si denique globorum, poſtquam ſuperior incidit in infer. & impetu in ſe invicem facto inferior acceleratur, ſuperior retardatur, deſiderantur loci ubi inter cadendum diſtantiam datæ rectæ æqualem acquirent: Quære primo locum ubi ſuperior impingit in infer. Dein ex cognitis tum magnitudinibus globorum, tum eorum ubi in ſe impingunt celeritatibus quære celeritates quas proxime poſt reflexionem habebunt, idque per modum Prob. XII. pag. 67. Poſtea quære loca ſumma ad quæ globi celeritatibus hiſce aſcenderent ſi ſurſum ferantur & inde cognoſces ſpatia quæ globi datis temporibus poſt reflexionem cadendo deſcribent, ut & differentiam ſpatiorum: & viciſſim ex aſſumpta illa differentia per Analyſin regredietur ad ipſa ſpatia cadendo deſcripta.

Ut ſi globus ſup. incidit in infer. ad punctum Π, & poſt reflexionem celeritas ſuperioris deorſum tanta ſit, ut ſi ſurſum eſſet aſcendere faceret globum illum infer. per ſpatium ΠN, tum tempora quibus globus ſuper. viciſſim deſcenderet per ſpatia NΠ, NG & inferior per ſpatia MΠ & MH, forent ut $\sqrt{N\Pi}$, $\sqrt{NG}$, $\sqrt{M\Pi}$, $\sqrt{MH}$, adeoque tempora quibus globus ſup. conficeret ſpatium ΠG & in- TAB. V. FIG. 4.

 fer.

fer. ſpatium ΠH forent ut $\sqrt{NG} - \sqrt{N\Pi}$ ad $\sqrt{MH} - \sqrt{M\Pi}$. Pone hæc tempora æqualia eſſe & erit $\sqrt{NG} - \sqrt{N\Pi} = \sqrt{MH} - \sqrt{M\Pi}$. Et inſuper cum detur diſtantia GH pone $\Pi G + GH = \Pi H$. Et harum duarum æquationum reductione ſolvetur problema. Ut ſi.

Sit $M\pi = a$ $N\pi = b$ $GH = 2c$ $\pi G = x$	Erit juxta priorem æquationem $x + 2c = \Pi H$ & $x + 2c + M\Pi = MH = a + 2c + x$. Sed $\Pi G +$

$N\Pi = b + x = NG$. Ergo his inventis, juxta priorem æquationem erit $\sqrt{b+x} - \sqrt{b} = \sqrt{a+2c+x} - \sqrt{a}$. Hinc $\sqrt{a} - \sqrt{b} = \sqrt{a+2c+x} - \sqrt{b+x}$. Et $a - 2\sqrt{ab} + b = a + 2c + x + b + x - 2\sqrt{(a+2c+x) \times (b+x)}$. Et $x + c + \sqrt{ab} = \sqrt{(ab + 2bc + bx + ax + 2cx + xx)}$. Et $x^2 + c^2 + ab + 2cx + 2x\sqrt{ab} + 2c\sqrt{ab} = ab + 2bc + bx + ax + 2cx + xx$. Et $2x\sqrt{ab} - bx - ax = 2bc - cc$. Et $x = (2bc - cc) : (2\sqrt{ab} - a - b)$ Q. E. I.

PROB. LIV.

Si duo globi ſint A, B *quorum ſuperior* A *ab altitudine* G *decidens in alterum inferiorem* B *a fundo* H *verſus ſuperiora reſilientem*

tem incidat, & hi globi ita per reflexionem ab invicem denuo recedant ut gl. A *vi reflexionis illius ad altitudinem priorem* G *redeat, idque eodem tempore quo gl.* B *inferior ad fundum* H *revertitur: dein gl.* A *rurſus decidat & in gl.* B *à fundo reſilientem denuo incidat, idque in eodem loco* AB *ubi prius in ipſum incidebat; & ſic perpetuo globi ab invicem reſiliant rurſusque ad eundem locum redeant: Ex datis globorum magnitudinibus, poſitione fundi, & loco* G *a quo globus ſuperior decidit invenire locum ubi globi in ſe mutuo impingent?*

Sit *e* centrum globi A & *f* centrum globi B, *d* centrum loci G in quo gl. A in maxima eſt altitudine, *g* centrum loci B ubi in fundum impingit, *a* ſemidiameter globi A, *b* ſemidiam. globi B, *c* punctum contactus globorum in ſe mutuo impingentium & H punctum contactus globi B & fundi. Et celeritas globi A ubi impingit in B ea erit quæ generatur caſu globi ab altitudine *de*, adeoque ut $\sqrt{de}$. Hac eadem celerit. reflecti debet A verſus ſuperiora ut ad G redeat. Et gl. B eadem celerit. deorſum reflecti debet qua aſcenderat ut eodem tempore redeat ad fundum quo inde receſſerat. Ut autem

hæc duo eveniant, globorum motus inter reflectendum æquales eſſe debent. Motus autem ex globorum magnitudinibus & celeritatibus componuntur, adeoque quod fit ex globi unius mole & celeritate æquatur ei quod fit ex globi alterius mole & celeritate. Unde ſi factum ex unius globi mole & celerit. dividatur per molem alterius globi, habebitur celeritas alterius globi proxime ante & poſt reflexionem, ſeu ſub fine aſcenſus & initio deſcenſus. Erit igitur hæc celeritas ut $A\sqrt{de} : B$, ſeu cum globi ſint ut cubi radiorum ut $a^3\sqrt{de} : b^3$. Ut autem hujus celeritatis quadratum ad quadratum celeritatis globi A proxime ante reflexionem, ita altitudo ad quam B hac celeritate, ſi occurſu globi A in eum non impediretur, aſcenderet ad altitudinem ed à qua globus A deſcendit. Hoc eſt, ut $deAq : Bq$ ad de ſeu ut Aq ad Bq vel a^6 ad b^6: ita altitudo illa prior ad x, ſi modo pro altitudine poſteriore ed ponatur x. Ergo hæc altitudo, ad quam nimirum B ſi non impediretur aſcenderet, eſt $a^6x : b^6$. Sit ea fK. & ob $fK + fg = dH - ef - gH$ hoc eſt $P - x$ erit (ſi modo pro dato $dH - ef - gH$ ſcribas p, & x pro incognito de, & habebitur $Kg = a^6x : b^6, + p - x$. Unde celeritas globi B ubi decidit à K ad fundum, hoc eſt,

eſt, ubi decidit per ſpatium Kg, quod centrum ejus inter decidendum deſcriberet, erit ut $\sqrt{a^6x : b^6}, + p - x$. At globus ille decidit a loco Bcf ad fundum eodem tempore quo gl. A aſcendit à loco Ace ad ſummam altitud. d, aut viciſſim deſcendit à d ad locum Ace, & proinde cum gravium cadentium celeritates æqualibus temporibus æqualiter augeantur, celeritas globi B deſcendendo ad fundum tantum augebitur quanta eſt celeritas tota quam gl. A eodem tempore cadendo à d ad e acquirat vel aſcendendo ab e ad d amittat. Ad celeritatem itaque quam gl. B habet in loco Bcf adde celer. quam gl. A habet in loco Ace, & ſumma quæ eſt ut $\sqrt{de}, + a^3\sqrt{de} : b^3$ ſeu $\sqrt{x} + a^3\sqrt{x} : b^3$ erit celeritas gl. B, ubi is in fundum incidit. Ergo $\sqrt{x} + a^3\sqrt{x} : b^3 = \sqrt{a^6x : b^6}, + p - x$. & $x, + 2a^3x : b^3, + a^6x : b^6 = a^6 : b^6, + p - x$. & $(a^6 + pb^6) : (a^6 + 2a^3b^3 + 2b^6) = x = ed$. Sed $ed + ec = dc$. Ergo habetur punctum c in quo globi in ſe mutuo impingunt. Q. E. F.

PROB. LV.

Erectis alicubi terrarum tribus baculis ad horizontale planum in punctis A, B *&* C *perpendicularibus, quorum is qui in* A *ſit ſex*

pedum, qui in B *octodecim ped. & qui in* C *octo ped. exiſtente linea* AB *triginta trium pedum: contingit quodam die extremitatem umbræ baculi* A, *tranſire per puncta* B *&* C, *baculi autem* B *per* A *&* C *ac baculi* C *per punctum* A. *Quæritur declinatio ſolis & elevatio Poli, ſive dies locusque ubi hæc evenerint?*

Quoniam umbra baculi cujusque deſcripſit Conicam ſectionem, ſectionem nempe Coni radioſi cujus vertex eſt baculi ſummitas; fingam BCDEF eſſe hujusmodi curvam (ſive ea ſit Hyperbola, Parabola vel Ellipſis) quam umbra baculi A eo die deſcripſit, ponendo AD, AE, AF ejus umbras fuiſſe cum BC, BA, CA reſpective fuerint umbræ baculorum B & C. Et præterea fingam PAQ eſſe lineam meridionalem ſive axem hujus curvæ ad quam demiſſæ perpendiculares BM, CH, DK, EN & FL ſunt ordinatim applicatæ. Has vero ordinatim applicatas indefinite deſignabo littera y & axis partes interceptas AM, AH, AK, AN, & AL littera x. Fingam denique æquationem $aa \pm bx \pm cxx = yy$, ipſarum x & y relationem (*i. e.* naturam Curvæ) deſignare, aſſumendo aa, b & c tamquam cognitas ut ex analiſi tandem inveniantur.

Ubi

Ubi incognitas x & y poſui tantum x^2 & TAB. V. y^2 quia æquatio eſt ad Conicam ſectio- FIG. 6. nem, & ipſius y dimenſiones impares omiſi quia ipſa eſt ordinatim applicata ad axem. Signa autem horum b & c, quia indeterminata ſunt, deſignavi notula ± quam indifferentur pro + aut − uſurpo, & ejus oppoſitum ∓ pro ſigno contrario. At ſignum quadrati aa affirmativum poſui, quia baculum A umbras in adverſas plagas (C & F, B & E) projicientem concava pars curvæ neceſſario complectitur & proinde ſi ad punctum A erigatur perpend. Aβ, hoc alicubi occurret curvæ, puta in B, hoc eſt, ordinatim applicata y, ubi x nullum eſt, erit reale. Nam inde ſequitur quadratum ejus, quod in eo caſu eſt aa, affirmativum eſſe.

Hæc igitur æquatio $aa \pm bx \pm cxx = yy$ ſicut terminis ſuperfluis non referta, ſic nec reſtrictior eſt quam ut ad omnes hujus problematis conditiones ſe extendat, Hyperbolam, Ellipſim vel Parabolam quamlibet deſignatura, prout ipſorum aa, b, c valores determinabuntur, aut nulli forte reperientur. Quid autem valent, quibusque ſignis b & c debent affici & inde quænam ſit curva, ex ſequenti Analiſi conſtabit.

Analiſis pars prior

Cum umbræ ſint ut altitudines baculorum erit BC: AD = AB: AE (= 18: 6) = 3: 1. Item CA: AF (= 8: 6) = 4: 3. Quare.

Dic AM = r MB = s AH = t & HC = $\bot v$	Ob ſim. tr. AMB, ANE, & AHC, ALF erunt AN = $-r:3$. NE = $-s:3$. AL = $-3t:4$

& LF = $\top 3v:4$. Quarum Signa ſignis ipſarum AM, MB, AC, HC contraria poſui, quia tendunt ad contrarias plagas reſpectu puncti A à quo ducuntur, axisve PQ cui inſiſtunt. His autem pro x & y in æquatione fictitia $aa \top bx \top cxx = yy$, reſpective ſcriptis...

r & s dabunt $aa \top br \top crr = ss$.

$-r:3$ & $-s:3$ dab. $aa \top br:3 \bot crr:9 = ss:9$

t & $\bot v$ dabunt $aa \bot bt \bot ctt = vv$

$-t:4$ & $\top v:4$ dab. $aa \top 3bt:4 \bot 9ctt:16 = 9vv:16$.

Jam è prima & 2da harum, exterminando ss, ut obtineatur r, prodit $2aa:\bot b = r$. Unde patet $\bot b$ eſſe affirmativum. Item è tertia & quarta exterminando vv ut obtineatur t, prodit $a^2:3b = t$. Et ſcriptis inſuper $2a^2:b$ pro r in 1a, & $aa:$

3b

$3b$ pro t in 3^2, oriuntur $3a^2 \perp 4a^4c:bb = ss$ & $4aa:3 \perp a^4c:9bb = \upsilon\upsilon$.

Porro demiſſa $B\lambda$ perpend. in CH, erit BC : AD ($= 3:1$) $= B\lambda : AK = C\lambda : DK$. Quare cum ſit $B\lambda$ ($= AM - AH = r - t$) $= 5a^2 : 3b$, erit $AK = 5a^2 : 9b$ vel potius $= -5a^2 : 9b$. jtem cum ſit $C\lambda$ ($= CH \perp BM = \upsilon \perp s$) $= \sqrt{4a^2:3 \perp a^4c:9bb} \perp \sqrt{3a^2 \perp 4a^4c:b^2}$, erit DK ($= C\lambda : 3$) $= \sqrt{4a^2:27 \perp a^4c:81b^2} \perp \sqrt{aa:3 \perp 4a^4c:9bb}$. Quibus in æquatione $a^2 + bx \perp cx^2 = y^2$, pro AK ac DK ſive x & y reſpective ſcriptis, prodit $4a^2:9 \perp 25a^4c:81b^2 = \frac{13}{27}a^2 \perp 37a^4c:81b^2 \perp 2\sqrt{4a^2:27 \perp a^4c:81b^2} \times \sqrt{a^2:3 \perp 4a^4c:9b^2}$. Et per reductionem $-b^2 \top 4a^2c = \perp 2\sqrt{36b^4 \perp 51a^2b^2c + 4a^4cc}$. Et partibus quadratis iterumque reductis exit $0 = 143b^4 \perp 196a^2b^2c$, ſive $-143b^2 : 196a^2 = \perp c$. Unde conſtat $\perp c$ negativam eſſe, adeoque æquationem fictitiam $a^2 \perp bx \perp cx^2 = y^2$ hujus eſſe formæ $a^2 + bx - cx^2 = y^2$ & ideo curvam quam deſignat eſſe Ellipſim. Ejus vero centrum & axes duo ſic eruuntur.

Ponendo $y = 0$, ſicut in figuræ verticibus P & Q contingit, habebitur $a^2 + bx = cx^2$ & (15) $x = b:2c \pm \sqrt{b^2:4c^2, + a^2:c} = AQ : AP$. Adeo-

que

que ſumpto AV $= b : 2c$; erit V centrum Ellipſis & VQ vel VP ($= \sqrt{bb : 16c^2, + a^2 : c}$) ſemiaxis maximus. Si porro ipſius AV valor $b \div 2c$ pro x in æquatione $aa + bx - cx^2 = y^2$ ſcribatur, fiet $aa, + b^2 : 4c = y^2$. Quare eſt $a^2, + b^2 : 4c = VZq$, hoc eſt quadrato ſemiaxis minimi. Denique in valoribus ipſarum AV, VQ, VZ jam inventis, ſcripto $143b^2 : 196a^2$ pro c, exeunt $98aa : 143b =$ AV, $112aa\sqrt{3} : 143b =$ VQ & $8a\sqrt{3} : \sqrt{143} =$ VZ.

Analiſeos pars altera.

Supponatur jam baculum puncto A inſiſtentem eſſe AR & erit RPQ planum meridionale, ac RPZQ conus radioſus cujus vertex eſt R. Sit inſuper TXZ planum ſecans Horizontem in VZ, ut & meridionale planum in TVX, quæ ſectio ſit ad axem mundi conive perpendicularis, & ipſum planum TXZ erit ad eundem axem perpendiculare & conum ſecabit in peripheria circuli TZX quæ ab ejus vertice pari ubique intervallo RX, RZ, RT diſtabit. Quare ſi PS ipſi TX parallela ducatur, fiet RS $=$ RP ob RX $=$ RT; nec non SX $=$ XQ ob PV $=$ VQ. Unde eſt RX vel RZ

TAB. V. FIG. 7.

(=

$(= \frac{1}{2} RS + \frac{1}{2} RQ) = \frac{1}{2} RP + \frac{1}{2} RQ$. Denique ducatur RV & cum VZ perpendiculariter inſiſtat plano RPQ (ſectio utique exiſtens planorum eidem perpendiculariter inſiſtentium) fiet tr. RVZ rectangulum ad V. Dictis jam.

$RA = \sqrt{d}$	Erit $AP = \sqrt{f} - \sqrt{e}$ &
$AV = \sqrt{e}$	$RP = \sqrt{f - 2\sqrt{ef} + e + d}$.
$VP = \sqrt{Q} = \sqrt{f}$	Item $AQ = \sqrt{f} + \sqrt{e}$ &
$VZ = \sqrt{g}$	$RQ = \sqrt{f + 2\sqrt{ef} + e + d}$.

adeoque $RZ\ (= \frac{1}{2} RP + \frac{1}{2} RQ) = \frac{1}{2} \sqrt{f - 2\sqrt{ef} + e + d} + \frac{1}{2}\sqrt{f + 2\sqrt{ef} + e + d}$. Cujus quadratum $\frac{1}{2}(d + e + f) + \frac{1}{2}\sqrt{(f^2 + d^2 + e^2 + 2df + 2de - 2ef)} = (RVq + VZq = RAq + AVq + VZq) = d + e + g$. Ergo $\sqrt{(f^2 + d^2 + e^2 + 2df + 2de - 2ef)} = d + e + 2g - f$. Et $f^2 + d^2 + e^2 + 2df + 2de - 2ef = d^2 + e^2 + 4g^2 + f^2 + 2de + 4dg - 2df + 4eg - 2ef - 4gf$. Et $df : g = g + d + e - f$. Denique $6,\ 98aa : 143b,\ 112a^2\sqrt{3} : 143b$ & $8a\sqrt{3} : \sqrt{143}$. (valoribus ipſorum AR, AV, VQ, & VZ pro $\sqrt{d}$, $\sqrt{e}$, $\sqrt{f}$ ac $\sqrt{g}$ reſtitutis, oritur $36, -196a^4 : 143b^2 + 192a^2 : 143 = 36 \times 14 \times 14a^2 : 143b^2$ & inde per reductionem $(49a^4 + 36 \times 49a^2) : (48aa + 1287) = bb$.

In Fig. 6. eſt $AMq + MBq = ABq$; hoc eſt

eſt $r^2 + s^2 = 33 \times 33$. Erat autem $r = 2a^2 : b$, & $s^2 = 3a^2, -4a^4 c : b^2$, & (ſubſtituto $143b^2 : 196a^2$ pro c) $s^2 = 4a^2 : 49$. Quare $4a^4 : b^2 + 4a^2 : 49 = 33 \times 33$. & inde per reductionem reſultat iterum $4 \times 49a^4 : (53361 - 4a^2) = bb$ Cujus partibus multiplicatis in crucem, ordinatis, ac diviſis per 49, exit $4a^4 = 981a^2 + 39204$, cujus radix $a^2 = (981 + \sqrt{1589625}) : 8 = 280,2254144$.

Supra inventum fuit $(4 \times 49a^4) : (53361 - 4a^2) = b^2$, ſive $14a^2 : \sqrt{53361 - 4a^2} = b$. Unde $AV = (98a^2 : 143b)$ eſt $7\sqrt{53361 - 4a^2} : 143$, & VP vel $VQ = (112a^2\sqrt{3} : 143b)$ eſt $8\sqrt{160083 - 12a^2} : 143$. Hoc eſt ſubſtituendo 280, 2254144 pro a^2, ac terminos in decimales numeros reducendo, $AV = 11,188297$, & VP vel $VQ = 22,147085$. Adeoque AP $(= PV - AV) = 10,958788$, & AQ $(= AV + VQ) = 33,335382$.

Denique ſi $\frac{1}{6}$ AR ſeu 1 ponatur radius, erit $\frac{1}{6}$ AQ ſeu 5,555897 tangens anguli ARQ 79 *gr.* 47′. 48″. & $\frac{1}{6}$ AP ſeu 1,826465 tangens anguli ARP 61. *gr.* 17′ 57″. Quorum angulorum ſemiſumma 70 *gr.* 32′. 52″ eſt complementum declinationis ſolis, & ſemidifferentia 9 *gr.* 14′. 56″, complementum latitudinis loci.

Pro-

Proinde declinatio ſolis erat 19 *gr.* 27′. 8″, & latitudo loci 80 *gr.* 45′. 4″. Quæ erant invenienda.

PROB. LVI.

E Cometæ motu uniformi rectilineo per cælum trajicientis locis quatuor obſervatis, diſtantiam a terra, motusque determinationem in Hypotheſi Copernicæa colligere? TAB. V. FIG. 8.

Si è centro Cometæ in locis quatuor obſervatis, ad planum Eclipticæ demittas totidem perpendicula; ſintque A, B, C, D puncta in plano illo in quæ perpendicula incidunt; per puncta illa age rectam AD & hæc ſecabitur à perpendiculis in eadem ratione cum linea quam Cometa motu ſuo deſcribit, hoc eſt, ita ut ſit AB ad AC ut tempus inter 1^am^ & 2^dam^ obſervationem ad tempus inter 1^am^ ac 3^am^, & AB ad AD ut tempus illud inter 1^am^ & 2^am^ ad tempus inter 1^am^ & 4^tam^. Ergo ex obſervationibus dantur rationes linearum AB, AC, AD ad invicem.

Inſuper in eod. Eclipticæ plano ſit S Sol, EH arcus lineæ Eclipticæ in qua terra movetur, E, F, G, H loca quatuor terræ temporibus obſervationum, E locus 1us, F 2^dus^, G 3^tius^, H 4^tus^. Junge

ge AE, BF, CG, DH & produc donec tres poſteriores priorem ſecent in I, K, & L; BF in I, CG in K, DH in L. Et erunt anguli AIB, AKC, ALD differentiæ longitudinum obſervatarum Cometæ; AIB differentia longitudinum loci primi Cometæ & 2^{di}; AKC diff. longit. loci primi & 3^{tii}; & ALD diff. longit. loci primi & 4^{ti}. Dantur itaque ex obſervationibus anguli AIB, AKC, ALD.

Junge SE, SF, EF; & ob data puncta S, E, F, datumque ang. ESF dabitur ang. SEF. Datur etiam ang. SEA, utpote differentia longitudinis Cometæ & Solis tempore obſerv. 1^{mæ}. Quare ſi complementum ejus ad duos rectos, nempe ang. SEI addas angulo SEF dabitur ang. IEF. Ergo tr. IEF dantur anguli una cum latere EF, adeoque datur etiam latus IE. Et ſimili argum. dantur KE & LE. Dantur igitur poſitione lineæ quatuor AI, BI, CK, DL, adeoque Problema huc redit, ut lineis quatuor poſitione datis, quintam invenias quæ ab his in data ratione ſecabitur.

Demiſſis ad AI perpend. BM, CN, DO ob datum ang. AIB datur ratio BM ad MI. Eſt & BM ad CN in data ratione BA ad CA, & ob datum ang. CKN datur ratio CN ad KN. Quare datur & ratio BM ad KN & inde ratio quoque BM

ad

ad MI−KN, id eſt, ad MN+IK. Cape P ad IK ut eſt AB ad BC, & cum ſit MA ad MN in ead. ratione, erit etiam P+MA ad IK+MN in ead. ratione, id eſt, in ratione data. Quare datur ratio BM ad P+MA. Et ſimili argumento, ſi capiatur Q ad IL in ratione AB ad BD dabitur ratio BM ad Q+MA. Et proinde ratio BM ad ipſorum P+MA & Q+MA differentiam, quoque dabitur. At differentia illa, nempe P−Q vel Q−P, datur. Et proinde dabitur BM. Dato autem BM ſimul dantur P+MA & MI & inde MA, ME, AE, & angulus EAB.

His inventis, erige ad A lineam plano Eclipticæ perpendic. quæ ſit ad lineam EA ut tangens latitudinis Cometæ in obſervatione prima ad radium, & iſtius perpend. terminus erit locus centri Cometæ in obſervatione prima. Unde datur diſtantia Cometæ a terra tempore illius obſervationis. Et eodem modo ſi è puncto B erigatur perpend. quæ ſit ad lineam BF ut tangens latitudinis Cometæ in obſervatione 2da ad radium, habebitur locus centri Cometæ in obſerv. illa 2da. Et acta linea a loco primo ad locum 2dum, ea eſt in qua Cometa per Cœlum trajicit.

PROB. LVII.

Si angulus datus CAD *circa punctum angulare* A, *poſitione datum*, & *angulus datus* CBD *circa punctum angulare* B, *poſitione datum, ea lege circumvolvantur ut crura* AD, BD *ad rectam poſit. datam* EF *ſeſe ſemper interſecent: Invenire lineam illam curvam quam reliquorum crurum* AC, BC *interſectio* C *deſcribit?*

TAB. V. FIG. 9. Produc CA ad *d* ut ſit A*d* = AD, & CB ad *δ* ut ſit B*δ* = BD. Fac. ang. A*de* = ang. ADE & ang. B*δf* = ang. BDF & produc AB utrimque donec ea occurrat *de* & *δf* in *e* & *f*. Produc etiam *ed* ad G, ut ſit *d*G = *δf*, & à puncto C ad lineam AB ipſi *ed* parallelam age CH & ipſi *fδ* parallelam CK. Et concipiendo lineas *e*G *fδ* immobiles manere dum ang. CAD, CBD lege proſcripta circa polos A & B volvantur, ſemper erit G*d* = *fδ* & tr. CHK dabitur ſpecie. Dic itaque.

A*e* = *a*	Erit BK : CK = B*f* : *fδ*. Er-
*e*G = *b*	go *fδ* = *cy* : *x* = G*d*. & G*e*−G*d*
B*f* = *c*	= *ed* = *b*, − *cy* : *x*. Cum detur
AB = *m*	ſpecie tr. CHK, pone CK :
BK = *x*	CH = *d* : *e*; & CH : HK =
CK = *y*	*e* : *f*, & erit CH = *ey* : *d*,

&

& HK $= fy : d$. adeoque AH $= m - x, -fy : d$. Eſt autem AH : HC $= Ae : ed$, hoc eſt $(m - x, -fy : d) : ey : d = a : (b, -cy : x)$. Hinc (Ax. IV.) fit $fcy^2 + (dc - ae - fb)\, xy - dcmy - bdx^2 + bdmx = 0$. Ubi cum x & y ad duas tantum dimenſiones aſcendant patet curvam quam punctum C deſcribit eſſe Conicam ſectionem. Pone $(ae + fb - dc) : c = 2p$ & extracta radice fiet $y = px : f, + dm : 2f \pm \sqrt{(p^2x^2 : f^2, + bdx^2 : fc, + pdmx : f^2, - bdmx : fc, + d^2m^2 : 4f^2)}$. Unde Curva erit Hyperbola, ſi ſit $bd : fc$ affirmativum vel negativum & minus quam $p^2 : f^2$; Parabola, ſi ſit $bd : fc$ negativum & æquale p^2, f^2; Ellipſis vel circulus, ſi ſit $bd : fc$ negat. & majus quam $p^2 : f^2$. Q. E. I.

PROB. LVIII.

Parabolam deſcribere quæ per data quatuor puncta tranſibit.

Sint puncta illa data A, B, C, D. Junge AB & eam biſeca in E. Et age rectam aliquam VE, quam concipe diametrum eſſe Parabolæ, puncto V exiſtente vertice ejus. Junge AC ipſique AB parallelam age DG occurrentem AC in G. TAB. VI. FIG. 1.

Dic AB $= a$ | In AC cape AP cujusvis
AC $= b$ | longit. & à P age PQ pa-
AG $= c$ | rallelam AB, & concipi-
GD $= d$ | endo Q punctum eſſe Pa-
AP $= x$ | rabolæ, æquationem quam-
PQ $= y$ | cumque ad Parabolam

aſſume quæ relationem inter AP & PQ exprimat. Ut quod ſit $y = e + fx \pm \sqrt{gg + hx}$.

Jam ſi ponas AP ſeu $y = 0$, puncto P incidente in ipſum A, fiet PQ ſeu $y = 0$, ut & $= -$ AB. Scribendo autem in æquat. aſſumpta 0 pro x, fiet $y = e \pm \sqrt{gg} = e \pm g$. Quorum valorum ipſius y major $e + g$ eſt 0, minor $e - g = -$ AB ſive $-a$. Ergo $e = -g$ & $e - g$, hoc eſt $-2g = -a$, ſive $g = \frac{1}{2}a$. Adeoque vice æquat. aſſumptæ habebitur hæc $y = -\frac{1}{2}a + fx \pm \sqrt{\frac{1}{4}aa + bx}$.

Ad hæc ſi ponas AP ſeu $x =$ AC, ita ut punctum P incidat in C, fiet iterum PQ $= 0$. Pro x igitur in æquatione noviſſima ſcripto b & pro y, 0, vice aſſumptæ æquationis erues (Axiom. IV) hanc $y = -\frac{1}{2}a + fx \pm \sqrt{(\frac{1}{4}aa + ffbx - fax)}$.

Inſuper ſi ponas $x = c$ fiet $y = -d$. Quare pro x & y in æquat. noviſſ. ſcribe c & $-d$, Et (Axio. IV.) fiet $ff = 2df : (b - c), + (dd - ad) : (bc - cc)$. Pro $b - c =$ GC ſcri-

be

be k, fiet æquatio cujus radix $f = d : k$, $\pm \sqrt{(ddc + ddk - adk) : kkc}$. Invento autem f, æquatio ad parabolam, nempe $y = -\frac{1}{2} a + fx \pm \sqrt{(\frac{1}{4} aa + ffbx - fdx)}$ plene determinatur. Cujus itaque conſtructione Parabola etiam determinabitur. Conſtructio autem eſt hujusmodi. Ipſi BD parallelam age CH, occurrentem DG in H. Inter DG ac DH cape med. prop. DK, & ipſi CK age parall. EI biſecantem AB in E & occurrentem DG in I. Dein produc IE ad V, ut ſit. EV : EI = EBq : (DIq − EBq) & erit V vertex, VE diameter & BEq : VE latus rectum Parabolæ quæſitæ,

PROB. LIX.

Conicam ſectionem per data quinque puncta deſcribere?

Sint puncta iſta A, B, C, D, F. jungeTAB. VI. FIG. 2. AC, BE ſe mutuo ſecantes in H. Age DI parall. BE & occurrentem AC in I. Item EK parall. AC & occur. DI productæ in K. Produc ID ad F & EK ad G; ut ſit AHC : BHE = AIC : FID = EKG : FKD & erunt puncta F ac G in conica ſectione, ut notum eſt. Nota tamen quod ſi punctum H cadat inter puncta omnia

A, C & B, E, vel extra ea omnia, punctum I cadere debebit vel inter puncta omnia A, C & F, D, vel extra ea omnia; & punctum K inter omnia D, F & E, G, vel extra ea omnia. At ſi punctum H cadit inter duo puncta A, C & extra alia duo B, E vel inter illa duo B, E & extra altera duo A, C, debebit punctum I cadere inter duo punctorum A, C & F, D, & extra alia duo eorum; & ſimiliter punctum K cadere debebit inter duo punctorum D, F & E, G & extra alia duo eorum: id quod fiet capiendo IF, KG, ad hanc vel illam partem punctorum I, K pro exigentia problematis. Inventis punctis F ac G biſeca AC, EG in N & O; item BE, FD in L & M junge NO, LM ſe mutuo ſecantes in R; & erunt LM & NO diametri conicæ ſectionis, R centrum ejus, & BL, FM ordinatim applicatæ ad diametrum LM. Produc LM hinc inde ſi opus eſt ad P & Q, ita ut ſit $BLq : FMq = PLQ : PMQ$ & erunt P & Q vertices conicæ ſect. & PQ latus transverſum. Fac $PLQ : LRq = PQ : T$, & erit T latus rectum. Quibus cognitis cognoſcitur Figura.

Reſtat tantum ut doceamus quomodo LM hinc inde producenda ſit ad P & Q ita ut fiat $BLq : FMq = PLQ : PMQ$.

Nempe

Nempe PLQ ſeu PL x LQ eſt (PR-LR) x (PR+LR) = PRq-LRq (§. 25), nam PL eſt PR-LR & LQ eſt RQ+LR ſeu PR+LR. Et ad eundem modum PMQ = PRq-RMq. Ergo BLq : FMq = (PRq-LRq) : (PRq-RMq) & *dividendo* (BLq-FMq) : FMq = (RMq-LRq) : (PRq-RMq). Quare cum dentur BLq-FMq; FMq & RMq-LRq dabitur PRq-RMq. Adde datum RMq & dabitur ſumma PRq, adeoque & latus ejus PR, cui QR æqualis eſt.

PROB. LX.

Conicam ſectionem deſcribere quæ tranſibit per quatuor data puncta, & in uno iſtorum punctorum continget rectam poſitione datam?

Sint puncta quatuor data A, B, C, D & recta poſit. data AE quam conica ſectio contingat in puncto A. Junge duo quævis puncta D, C & DC producta ſi opus eſt occurrat tangenti in E. Per quartum punctum B ipſi DC age parall. BF, quæ occurrat eidem tangenti in F. Item tangenti parallelam age DI, quæ occurrat ipſi BF in I. In FB, DI, ſi opus eſt productis, cape FG, HI ejus longitudinis ut ſit AEq:CED = AFq : BFG = DIH : BIG. Et erunt puncta G & H in Conica ſect. ut notum eſt; ſi modo ca- TAB. VI. FIG. 3.

pias FG, IH ad legitimas partes punctorum F & I, juxta regulam in prob. ſuper. traditam. Biſeca BG, DC, DH, in K, L & M. Junge KL, AM ſe mutuo ſecantes in O, & erit O centrum, A vertex, & HM ordin. applicata ad ſemidiametrum AO. Quibus notis cognoſcitur figura.

PROB. LXI.

Conicam ſectionem deſcribere quæ tranſibit per tria data puncta, & in duobus iſtorum punctorum continget rectas poſitione datas?

TAB. VI. FIG. 4. Sint puncta illa data A, B, C, Tangentes AD, BD ad puncta A & B; D communis interſectio tangentium. Biſeca AB in E. Age DE & produc eam donec in F occurrat CF actæ parallelæ AB; & erunt DF diameter & AE, CF ordin. applicatæ ad diametrum. Produc DF ad O & in DO cape OV med. prop. inter DO & EO, ea lege ut ſit etiam AE*q* : CF*q* = VE x (VO + OE) : VF x (VO + OF); & erit V vertex & O centrum Figuræ. Quibus notis figura ſimul cognoſcitur. Eſt autem VE = VO - OE. Hinc VE x (VO + OE) = (VO - OE) x (VO + OE) = VO*q*

$= VOq - OEq$. Præterea quia VO med. prop. eſt inter DO & EO erit $VOq = DOE$, adeoque $VOq - OEq = DOE - OEq = DEO$. Et ſimili argum. erit $VF \times (VO + OF) = VOq - OFq = DOE - OFq$. Ergo $AEq : CFq = DEO : (DOE - OFq)$ Sed $OFq = EOq - 2FEO + FEq$. Ergo $DOE - OFq = DOE - OEq + 2FEO - FEq = DEO + 2FEO - FEq$. Et $AEq : CFq = DEO : (DEO + 2FEO - FEq) = DE : (DE + 2FE, - FEq : EO)$. Datur ergo $DE + 2FE, - FEq : EO$. Aufer hoc de dato $DE + 2FE$, & reſtabit $FEq : EO$ datum. Sit illud N; & erit $FEq : N = EO$, adeoque dabitur EO quo ſimul datur VO med. prop. inter DO & EO.

Hoc modo per Theoremata quædam Apollonii ſatis expedite reſolvimus tria hæc ultima problemata quæ tamen ſine iſtis theorematibus per Algebram ſolam reſolvi poſſent. Confer *Arith. Univ.* pag. 175 ad 178. ſed tempus eſt jam æquationum reſolutionem docere. Nam poſtquam problema ad æquationem finalem deductum eſt, (32) radices illius æquationis quæ quantitates ſunt Problemati ſatisfacientes extrahere opportebit. (vide Prob. XXIX pag. 107.)

RADICES IMPOSSIBILES.

44 „ In omnibus æquationibus augen-
„ do vel minuendo terminos earum, ex in-
„ æqualibus radicibus duæ primo æquales,
„ dein impoſſibiles evadere ſolent, inde
„ fit quod radicum impoſſ. numerus ſem-
„ per ſit par " (15 Cor. 2.)

Hæc $x^2 - 5x + 6 = 0$ minuendo unitate ſummam 5, vel augendo unitate productum 6; fit $x^2 - 4x + 6 = 0$, vel $x^2 - 5x + 7 = 0$, quarum radices ſunt impoſſibiles. Econtra $x^2 - 6x + 6 = 0$, vel $x^2 - 5x + 5 = 0$, radices quidem ſurdas ſed reales habent. (15) Hinc (ſi quid video) fallit Regula Cl. Maclaurin, ad calcem Arith. univ. pag. 318, ubi docet in hac æquatione $x^4 - 4ax^3 + 6a^2x^2 - 4ab^2x + b^4 = 0$ quatuor dari radices imaginarias, ſi a ſit majus quam b. Nam in ea poſito $b = a$ fiunt quatuor radices æquales. Ergo poſito b minus quam a evadent hæ inæquales, non vero impoſſibiles?

THEOR. I „ Factus cujusque paris ra-
„ dicum impoſſibilium eſt ſemper poſitiva
„ quantitas." nam $(a + \sqrt{-b}) \times (a - \sqrt{-b}) = a^2 + b^2$. Sed factus cujusque paris ſurdarum realium eſt quantitas poſitiva, ſi ſemiſumma a major eſt quam ſemidifferentia $\sqrt{b}$, ſi minor erit negativa (25).

45.

45. *Corol.* „ Omnis æquatio dimenſio-
„ num parium cujus terminus ultimus eſt
„ ſub ſigno – duas ſaltem radices reales ha-
„ bet poſitivam unam, & alteram nega-
„ tivam". Hinc examinata *ead. pag.* 318.
æquatio $x^4 + 5x^3 - 6x^2 - x - 12 = 0$,
duas ſaltem radices reales habet. An re-
liquæ duæ ſint impoſſibiles? Affirmat *ibid.*
Cl. Maclaurin, quem confer.

THEOR. II. „ Omnis æquatio in qua
„ $pp - 2q$ eſt quantitas negativa duas
„ ſaltem radices impoſſibiles habet "
(25 Th. IV.).

46. Verum quot radices impoſſibiles
ſunt cognoſci fere poteſt per hanc Newto-
ni Regulam, quæ nunquam deficit niſi in
caſu quo radices omnes æquales ſunt:
Quare verbum ſuperaddam quo univer-
ſalis evadat.

„ Conſtitue ſeriem fractionum quo-
„ rum denominatores ſunt numeri in hac
„ progreſſione 1, 2, 3, 4, 5 &c. per-
„ gendo ad numerum usque qui eſt dimen-
„ ſionum æquationis; numeratores verò
„ eadem ſeries numerorum ordine con-
„ trario: divide unamquamque fractio-
„ nem poſteriorem per priorem. Fractio-
„ nes prodeuntes colloca ſuper terminis
„ mediis æquationis. Et ſub quolibet me-
„ diorum terminorum, ſi quadratum ejus

„ ductum

„ ductum in fractionem capiti imminen-
„ tem ſit *æquale* vel majus rectangulo
„ terminorum utrinque conſiſtentium,
„ colloca ſignum +; ſin minus ſignum −.
„ ſub primo vero & ultimo colloca ſig-
„ num +. Et tot erunt radices impoſſibi-
„ les quot ſunt in ſubſcriptorum ſignorum
„ ſerie mutationes *de* + in − & − in +."

Ut ſi habeatur æquatio $x^3 + px^2 + 3p^2x - q = 0$. Divido ſeriei hujus $\frac{3}{1}$, $\frac{2}{2}$, $\frac{1}{3}$ fractionum ſecundam $\frac{2}{2}$ per primam $\frac{3}{1}$ & tertiam $\frac{1}{3}$ per ſecundam $\frac{2}{2}$ & fractiones prodeuntes $\frac{1}{3}$ & $\frac{1}{3}$ colloco ſuper mediis terminis æquationis, ut vides.

$$\begin{array}{cccc} & \frac{1}{3} & \frac{1}{3} & \\ x^3 + & px^2 + & 3p^2x - & q = 0 \\ + & - & + & + \end{array}$$

Dein ſub px^2 ſcribo −, ob $p^2 \times \frac{1}{3} < 3p^2 \times 1$. Sub $3p^2x$ ſcribo +, ob $9p^4 \times \frac{1}{3} > -qp$. Sed ſub x^3 & $-q$ ſcribo +. Et ſignorum ſubſcriptorum quæ in hac ſunt ſerie + − + + mutationes duæ, una de + in −, alia de − in + indicant duas eſſe radices impoſſibiles.

Demonſtratio regulæ Auctoris. „ Si radicum ſumma ſecaretur in partes æquales, foret quadratum cujuslibet termini medii, ductum in fractionem capiti imminentem, rectangulo terminorum utrinque conſiſtentium æquale, majus autem ſi in partes inæquales ſecaretur.

Ergo

Ergo in nulla æquatione cujus radices omnes ſunt reales tale quadratum eſſe poteſt rectangulo *minus* & ubi hoc contingit tot ſunt radices impoſſibiles quot ſunt in ſubſcriptorum ſignorum ſerie mutationes de + in − & − in +. Q. E. D.

Hinc radicum impoſſibiles duæ ſunt...

$$\begin{array}{lccccc} & & \frac{1}{3} & \frac{1}{3} & & \\ \text{In} \ldots & x^3 & -4x^2 & +4x & -6 & =0 \\ & + & + & - & + & \end{array}$$

$$\begin{array}{lcccccc} & & \frac{3}{8} & \frac{4}{6} & \frac{3}{8} & & \\ \&\ \text{in}\ . & x^4 & -4x^3 & -6x^2 & -3x & -2 & =0 \\ & + & + & + & - & + & \end{array}$$

quippe in hac ultima $16 \times \frac{3}{8} > -6 \times 1$...

$$\begin{array}{lccccccc} & & \frac{2}{5} & \frac{1}{2} & \frac{1}{2} & \frac{2}{5} & & \\ \&\ \text{in}\ . & x^5 & -4x^4 & +4x^3 & -2x^2 & -5x & -4 & =0 \\ & + & + & - & + & + & + & \end{array}$$

Ubi termini duo vel plures ſimul deſunt, ſub primo terminorum deficientium collicandum eſt ſignum −, ſub ſecundo +, ſub tertio − & ſic deinceps variando, niſi quod ſub ultimo terminorum ſimul deficientium ſemper collocandum eſt ſignum +, ubi termini deficientibus utrinque proximi habent ſigna contraria. Ut in æquationibus

$$\begin{array}{ccccccc} x^5 & +ax^4 & * & * & * & +a^5 & =0 \\ + & + & - & + & - & + & \end{array}$$

&

$$\begin{array}{cccccc} x^5 & +ax^4 & * & * & * & -a^5 \\ + & + & - & + & - & + \end{array}$$

quarum prior quatuor, poſterior duas habet radices

ces impoſſibiles. Sic & $x^7-2x^6+3x^5-2x^4$
$\quad + \quad - \quad + \quad -$

$+x^3$ * * $-3=0$ ſex habet impoſſibiles.
$+ \quad - \quad + \quad +$

Hinc etiam cognoſci poteſt an radices impoſſibiles inter affirmativas radices lateant, an inter negativas. Nam ſigna terminorum ſignis ſubſcriptis variantibus imminentium indicant tot affirmativas eſſe impoſſibiles quot ſunt ipſorum variationes & tot negativas quot ſunt eorum ſucceſſiones ſine variatione. Sic in æquatione $x^5-4x^4+4x^3-2x^2-5x-4=0$, quo-
$+ \quad + \quad - \quad + \quad + \quad +$

niam ſignis ſubſcriptis $+--+$ imminent ſigna $-+-$ quæ indicant duas affirmativas radices, ideo radices duæ impoſſibiles inter affirmativas latebunt. Cum itaque omnium terminorum æquationis ſigna $+-+---$ indicent (34. Th. IV.) tres affirmativas radices & reliquas duas negativas & inter affirmativas lateant duæ impoſſibiles, ſequitur æquationis unam eſſe radicem vere affirmativam, duas negativas ac duas impoſſibiles. Quod ſi æquatio fuiſſet $x^5-4x^4-4x^3-2x^2-5x-4=0$
$+ \quad + \quad - \quad - \quad + \quad +$

tunc termini ſubſcriptis variantibus $+-$ imminentes indicant per ſigna ſua non variantia $--$ unam ex negativis radi-

cibus

cibus impoſſibilem eſſe, & termini ſignis − + imminentes, ſcilicet $-2x^2-5x$ per ſigna ſua − − indicant aliam ex negativis radicibus impoſſibilem eſſe. Hinc cum æquationis ſigna + − − − − − − indicent unam affirmativam radicem, cæteras quatuor negativas eſſe, ſequitur unam eſſe affirmativam, duas negativas ac duas impoſſibiles. Atque ita tot ſunt ſemper impoſſibiles radices quot regula correcta detegit; ſed non ſemper regula detegit tot quot ſunt impoſſibiles radices.

Novam methodum quæ applicata exemplis ſuperioribus ſuccedit, Cel. Campbell dedit *ad calcem Arith. Univ.* à pag. 333. ad *ultimam* 344; nempe loco fractionum in regula Newtoni capitibus imminentium, præſcribit poteſtatis perfectæ ejuſdem gradus ac eſt æquatio propoſita, uncias duplicatas pro denominatoribus eaſdemque unitate mulctatas pro numeratoribus reſpective notari, cæteraque ut prius fieri. Quo pacto detegit ſex radices impoſſibiles in æquatione $x^7-5x^6+15x^5-23x^4+18x^3+10x^2-28x+24=0$, ſc. $1+\sqrt{-3}$, $1-\sqrt{-3}$; $1+\sqrt{-2}$, $1-\sqrt{-2}$; $1+\sqrt{-1}$, $1-\sqrt{-1}$; (ſeptima eſt -1.) cum per regulam Newtoni duæ tantum radices impoſſibiles inveniantur. Verum in caſu quo radices propoſitæ ſunt æquales

les utraque methodus pariter deficit & nova corrigi nequit. Quippe quadratum unciæ cujuscumque intermediæ ductum ut vult Newtonus in fractionem capiti imminentem ſemper æquatur rectangulo unciarum utrimque conſiſtentium. Huc confer §. 10.

TRANSMUTATIONES ÆQUATIONUM.

47. „ Poſſumus ſupponere radicem æ-
„ quationis ex cognita & incognita ali-
„ qua quantitate utcumque componi &
„ perinde pro ea ſubſtituere id quod æ-
„ quipollens eſſe fingitur " tumque fit idem ac ſi in factoribus ſimplicibus illa ſubſtitutio facta fuiſſet. Ita $x^2-5x+6=0$ cujus radices ſunt 2 & 3, fingendo $x+1=y$ ſeu $x=y-1$, poſitoque in æquatione $y-1$ pro x & y^2-2y+1 pro x^2 transmutatur in $y^2-7y+12=0$, cujus radices quæ prius erant 2 & 3, jam ſunt 3 & 4. Evaderent autem 1 & 2 ſubſtituto $x+1$ pro x.

$$\begin{aligned} x^2 &= y^2-2y+1 \\ -5x &= \quad -5y+5 \\ +6 &= \qquad\quad +6 \\ \hline & y^2-7y+12=0 \end{aligned}$$

Hoc modo in altioribus æquationibus augendo vel diminuendo radices, ſi quæ impoſſibiles ſunt, hæ aliquando facilius deteguntur quam prius. Sic in x^3-3a^2x-

$3a^3 = 0$. radices nullæ per præcedentem regulam impoſſibiles apparent; at ſi augeas radices quantitate a, ſcribendo $y-a$ pro x, fiet $y^3-3ay^2-a^3=0$ in qua jam per regulam duæ radices impoſſibiles detegi poſſunt.

48. „ Poſſumus etiam eadem operatio-
„ ne ſecundos terminos æquationum tol-
„ lere " ut ex $x^m + px^{m-1} + qx^{m-2}$ &c, ponendo $x = y - \frac{p}{m}$ & transmutando. Exemplum eſto, $x^3 - 4x^2 + 4x - 6 = 0$, & $x = y + \frac{4}{3}$

$$\begin{array}{rcl} x^3 & = & y^3 + 4y^2 + \frac{16}{3}y + \frac{64}{27} \\ -4x^2 & = & -4y^2 - \frac{32}{3}y - \frac{64}{9} \\ +4x & = & +4y + \frac{16}{3} \\ -6 & = & -6 \\ \hline \text{erit} & & y^3 * - \frac{4}{3}y - \frac{146}{27} = 0 \end{array}$$

„ Eadem methodo poteſt & tertius
„ æquationum terminus tolli." Proponatur æquatio $x^4 - 3x^3 + 3x^2 - 5x - 2 = 0$, finge $x = y - e$ & transmuta; æquationis novæ coefficiens termini tertii erit $6e^2 + 9e + 3 = 0$ per hypotheſim; cujus radices, (§. 15.) erunt -1, & $-\frac{1}{2}$. Ergo in propoſita pro x ſubſtitue $y + 1$ vel $y + \frac{1}{2}$ & fiet id quod deſideratur.

„ Poſſunt & radices æquationis per
„ datos numeros multiplicari vel dividi;
„ & hoc pacto termini æquationum di-
„ minui, fractiones & aliquando radica-
„ les quantitates tolli, id quod brevius
„ quam Auctoris methodo præſtamus
„ per (34 Th. VI.).

Eſto . . . . . $y^3 * - \frac{4}{3}y - \frac{146}{27} = 0$

Triplicando . . 1, 3, 9, 27

fit $y^3 * - 12y - 146 = 0$

ſubduplic. . . . 1, 2, 4 8

fit . . . . $y^3 - 3y - 18\frac{1}{4} = 0$

& hic ſi tandem inveniatur y, ejus valor ductus in $\frac{2}{3}$ erit x radix æquationis primo propoſitæ $x^3 - 4x^2 + 4x - 6 = 0$.

Poteſt & per idem Theorema omnis æquatio affecta fractis in non affectam transmutari (*def.* 8. & 9).

Eſto . . $mx^3 + px^2 + qx + r = 0$

erit . . . . . $x^3 \frac{+ px^2 + qx + r}{m} = 0$

X . . . . . . 1, m, m^2, m^3

& $x^3 + px^2 + mqx + m^2 r = 0$

Hinc $9y^3 - 2y + 1 = 0$ triplicando fit $y^3 - 6y + 9 = 0$.

Porro ut ex . . $x^3 * - 2x + \sqrt{3} = 0$, tolla-
tur radicalis.

X

x . . . . $1, \sqrt{3}, \quad 3, \quad 3\sqrt{3}$
fietque . . . x^3 * $-6x + 9 = 0$

Imo ex . . $\frac{1}{x^3} - \frac{6}{x} + 9 = 0$ fit (34. Th. VII.) $9x^3 - 6x^2 + 1 = 0$, & ſub triplicando . . $x^3 - 2x^2$ * $+ 3 = 0$.

ÆQUATIONUM LIMITES.

49. Omnes æquationis cujuscumque terminos nec fractos nec ſurdos habentis, radices non ſurdæ & facti ex binis, ternis &c. ſunt aliqui ex diviſoribus integris ultimi termini (32). Hinc ubi conſtat nullum ultimi termini diviſorem eſſe aut radicem æquationis aut factum ex binis, ternis &c. ſimul conſtabit nullam eſſe radicem, radicumve factum niſi qui ſit ſurdus. (36. 37) Quid tum?

,, Multiplicetur æquationis terminus
,, unusquisque per numerum dimenſio-
,, num ejus & dividatur factum per ra-
,, dicem æquationis, & ſic pergatur ſem-
,, per multiplicando per numeros unita-
,, te minores quam prius & factum di-
,, videndo per radicem, donec tandem
,, termini omnes deſtruantur quorum
,, ſigna diverſa ſunt a ſigno primi ſeu
,, altiſſimi termini, præter ultimum. Et
,, numerus ille erit omni affirmativa ra-

„ dice major, qui in terminis prodeun-
„ tibus ſcriptus pro radice, efficit eo-
„ rum qui ſingulis vicibus per multipli-
„ cationem producebantur aggregatum
„ ejusdem ſemper eſſe ſigni cum primo
„ ſeu altiſſimo termino æquationis.

Eſto. . . $x^5 - 2x^4 - 10x^3 + 30x^2 + 63x - 120 = 0$, ($= 46$ ſi $x = 2$).

erit. . . . $5x^4 - 8x^3 - 30x^2 + 60x + 63 = 79$.

$5x^3 - 6x^2 - 15x + 15 = 1$.

$5x^2 - 4x - 5 = 7$.

$5x - 2 = 8$.

Jam cum primus terminus x^5 ſit $+ x^5$ tento numerum qui ſcriptus in his productis efficiet ea omnia affirmativa eſſe. & quidem tentando 1 fit $5x - 2 = 8$, ſed $5x^2 - 4x - 5 = -4$ negativum. Quare limes erit major quam 1. Hinc tento 2 pro x & poſt ſubſtitutiones evadunt ſingula producta $= +$. Quare 2 erit major quam radicum affirmativarum maxima. Si limitem negativarum deſiderem tento numeros negativos, vel quod perinde eſt muto ſigna terminorum alternorum & tento affirmativos. Sed in ſecunda $5x^4 + 8x^3 - 30x^2 - 60x + 63$, in qua maxime prævalere videntur numeri negativi, pro x ſubſtituendo tam 1 quam 2 prodit $-$. Ergo limes major erit quam 2. Sed ſub-

ſti-

ſtituendo 3 reſultat + tam in eo producto quam in cœteris; id quod ſola inſpectione colligere licet. Quare-3 tranſcendit omnes radices negativas. Itaque habentur limites 2 & - 3. Nec opus eſt jam ultimi termini 120, diviſores qui ſunt numero 32 tentare, niſi qui ſunt inter hos limites nimirum 1, -1, & -2. Nam ſi horum nullus radix eſt, certum eſt æquationem non habere radicem, niſi quæ ſit ſurda.

DEMONSTRATIO Regulæ Auctoris. Omnes radices affirmativæ ſignum habent diverſum a primo in factoribus ſimplicibus æquationum. Ergo numerus ex cujus ſubſtitutione in his facta pro incognita, reſultat idem ſignum cum primo eſt omni affirmativa radice major(47).Sed eodem redit ſive numerum illum ſubſtituas in factoribus ſimplicibus ſingulis ſive in facto ex illis. Ergo numerus ex cujus ſubſtitutione pro incognita, tam in aggregatis reſultantibus quam in propoſita, reſultat idem ſignum cum primo, major eſt omni radice affirmativa. Q. E. D.

COR. Numeri reſultantes ſunt id quo ſubducto reſpective de aggregatis fiunt hæc = 0 & diviſibilia per numerum ſubſtitutum. Unde ſequitur per multiplicationem quam primo loco præſcribit Auctor augeri radices affirmativas adeoque

& minui negativas propoſitæ ita ut ſingulæ inter duos limites lateant. Sunt autem limites omnes necnon aggregata ſingula reſultantia $= 0$ dum radices omnes propoſitæ ſunt æquales.

Regula Cardani.

50. Reductio quæ fit per extractionem ſurdæ radicis quadraticæ vel cubicæ poſſibilis eſt interdum, ſed perparvi uſus. Sunt tamen reductiones quædam cubicarum æquationum vulgo notæ, quas ſi præterirem Lector fortaſſe deſideraret. Proponatur $x^3 - 3px^2 - 3qx - 2r = 0$. Pono $x = x + p$ ut (per 47) fiat $x^3 - 3qx - 2r = 0$ in qua pono $a + b$ pro x.

$$\begin{aligned} &\text{fitque} \ldots & x^3 &= a^3 + 3ab \times (a+b) + b^3 \\ &\& \ldots\ldots & -3qx &= \quad -3q \times (a+b) \\ &\& \ldots\ldots & -2r &= \qquad\qquad -2r \end{aligned}$$

In qua ſi $a^3 + b^3 - 2r = 0$, erit $3ab - 3q = 0$ & $ab = q$. & $a^3 b^3 = q^3$; ſed $a^3 + b^3 = 2r$. Ergo (per 15) $a^3 = r + \sqrt{r^2 - q^3}$ & $b^3 = r - \sqrt{r^2 - q^3}$ & $a + b = x = \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$. Q.E.I.

Exemplum. Eſto. $x^3 - 6x^2 + 6x + 12 = 0$. Ut tollatur ſecundus terminus pono $x =$

x

$x+2$ & orietur $x^3 * -6x+8=0$, ubi $-q=-2$ & $r=4$. Ergo $x=\sqrt[3]{4+\sqrt{8}}+\sqrt[3]{4-\sqrt{8}}$. Sed $x+2=x$. Ergo $x=2+\sqrt[3]{4+\sqrt{8}}+\sqrt[3]{4-\sqrt{8}}$ erit radix propoſitæ realis. Reliquæ duæ erunt idem valor ductus in $\frac{-1\pm\sqrt{-3}}{2}$, hoc eſt impoſſibiles. Cujus rei ratio eſt quod unitas cubica reſolvatur in tres radices ſcilicet 1, $\frac{-1+\sqrt{-3}}{2}$ & $\frac{-1-\sqrt{-3}}{2}$, ut patet ſi dividatur $y^3-1=0$ per $y-\sqrt[3]{1}$ ſeu per $y-1=0$.

Et hoc modo poſſunt erui radices omnium æquationum cubicarum ubi q affirmativum eſt vel etiam ubi q negativum eſt & ſimul $r^2 > q^3$, hoc eſt *ubi duæ ex radicibus ſunt impoſſibiles*. Quippe ſi fuerit transmutata $x^3 * + 3qx \pm 2r = 0$ erit $0-6q$ ſumma negativa quadratorum radicum (25 Th. IV.) quod impoſſibiles duas radices indicat, adeoque tertiam quæ realis eſt, rite exprimi par $a+b=x$. At ubi q negativum eſt & ſimul $q^3 > r^2$, fit $\sqrt{r^2-q^3}$ ſemidifferentia inter a^3 & b^3 quantitas impoſſibilis, adeoque hoc in caſu $x=a+b$

im-

impoſſibilis evadit, quia *tres ſunt radices poſſibiles & inæquales* quas ſingulas exprimere per $x = a + b$ impoſſibile eſt & ex hypotheſi impoſſibili, concluſionem impoſſibilem colligi mirum eſſe non debet. Addo *inæquales* textui Auctoris; quia ſi tres radices ſint poſſibiles & æquales, exprimi poterunt ſingulæ per $x = a + b$ & invenientur, ope Regulæ, ſequenti methodo qua ſimul liberamur a labore ſecundos æquationum terminos tollendi.

51. Si æquationis generalis $x^3 - 3qx - 2r = 0$, radix x transmutetur in $x - p$ ut redeat 2^{dus} terminus, ſublatus per $x + p$. *ſupra*...

$$\text{fiet} \ldots \left.\begin{array}{l} x^3 - 3px^2 + 3p^2x - p^2 \\ \qquad\qquad - 3qx + 3pq \\ \qquad\qquad\qquad - 2r \end{array}\right\} = 0$$

Cujus radix erit $x = p + \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$. (47.)

Hinc ſi proponatur $x^3 - 6x^3 + 6x + 12 = 0$ (ut ſupra) erit $p = 2$ (ob $-3p = -6$); $4 = 2$ (ob $3p^2 - 3q = 6$) & $r = 4$ (ob $-p^3 + 3pq - 2r = 12$) Ergo $x = 2 + \sqrt[3]{4 + \sqrt{8}} + \sqrt[3]{4 - \sqrt{8}}$. *ut ſupra.*

Si

Si proponatur $x^3-3x^2+3x-1=0$. cujus tres radices ſunt poſſibiles & æquales, erit $p=1$, $q=0$, & $r=0$. Ergo $x=1$.

Si proponatur $x^3-x^2-x+1=0$, cujus tres radices ſunt poſſibiles & in abſtracto ſaltem æquales, erit $p=\frac{1}{3}$; $q=\frac{4}{9}$; $r=-\frac{8}{27}$; & $r^2-q^3=0$; Ergo $x=p+2\sqrt[3]{5}=\frac{1}{3},+2 \times -\frac{2}{3}=-1$.

52. „ Poſſunt etiam æquationum bi- „ quadraticarum radices, mediantibus „ cubicis erui & exprimi" ſublato prius 2do termino. Ut ſi ſit æquatio $x_2+\sqrt{q}x^2+\sqrt{r}x+\sqrt{s}=0$. Hujus factores pone eſſe $x+\sqrt{e}x+\sqrt{f}=0$, & $x-\sqrt{e}x+\sqrt{g}=0$, hoc eſt eandem eſſe cum hac $x^2(+\sqrt{f}+\sqrt{g}-e)\sqrt{x^2}+(\sqrt{eg}-\sqrt{ef})\sqrt{x}+\sqrt{fg}=0$. & collatis terminis fiet $\sqrt{f}+\sqrt{g}-e=\sqrt{q}$, $\sqrt{eg}-\sqrt{ef}=\sqrt{r}$ & $\sqrt{fg}=\sqrt{s}$. Quare $\sqrt{f}+\sqrt{g}=\sqrt{q}+e$ & $\sqrt{g}-\sqrt{f}=\sqrt{r}:e$. Sed (per 15) $2\sqrt{g}=\sqrt{q}+e+\sqrt{r:e}$, & $2\sqrt{f}=\sqrt{q}+e-\sqrt{r:e}$ Ergo $4\sqrt{fg}=4\sqrt{s}=q+2e\sqrt{q}+e^2,-r:e$. Et $e^3+2e^2\sqrt{q}\times(q-4\sqrt{s})e,-r=0$, cujus inventa radice e (per 51 vel ſecus) facile ex reductis eruuntur $\sqrt{f}$ & $\sqrt{g}$, atque his valoribus ſubſtitutis in factoribus $x+\sqrt{e}x+\sqrt{f}=0$, & $x-\sqrt{e}x+\sqrt{g}$, habentur (per 15) quatuor radices propoſitæ: niſi tamen fuerint

quatuor poſſibiles & inæquales, quo in caſu tres radices æquationis cubicæ e^3 + &c. erunt poſſibiles & inæquales, adeoque per (50. 51.) extrahi nequeunt.

53. Poſtquam Auctor Æquationum proprietates, transmutationes, limites & omnis generis reductiones docuit, terminat opus per *Appendicem de æquationum conſtructione lineari.* Verum in Opuſculis Opuſc. 2. docet Methodum determinandi radices æquationum per ſeries infinitas, quæ univerſalior ſit nec opus habeat curvarum conſideratione, quod non ita videtur ſubjecto mere Algebraico convenire. Hanc ideo alteri ſubſtitutam hic invenire, ſpero, non diſplicebit. Sed quia Methodus pendet ab Analogia perpetua, quæ eſt inter decimales fractiones & Algebraicos terminos in infinitum productos, calculum decimalium præmitto.

Calculus Decimalium.

54. Fracti decimales ſunt numeri qui in ratione ſubdecupla perpetuo decreſcunt, non ſecus ac integri ab unitate perpetuo in ratione decupla creſcunt. Solent autem ab his illi interjecto commate diſtingui. Sic numerus 732,569 denotat ſeptingentas triginta duas unitates,

tes, unà cum quinque decimis, ſex centeſimis & novem milleſimis partibus unitatis. Et numerus 0,064 denotat ſex centeſimas & quatuor milleſimas partes unitatis.

Quæcumque fractio poteſt ad decimalem reduci; nam ut denominator reducendæ ad ſuum numeratorem ita ad ſuum denominator reductæ. Eſto reducenda $\frac{1}{7}$ & denominator reductæ 1000000 cujus numerator ſit x, eritque (poſito $\frac{1}{7}$ = 0,142857) error minor quam una millioneſima unius ſeptimæ, cum aliquid de numeratore negligatur, foretque nullus ſi in infinitum produceretur operatio.

„ Additio & ſubductio numerorum
„ decimalium perinde fit ac integrorum.

630,953	35,72	46,5003	308,7
51,0807	14,32 *ſubd.*	3,078 *ſubd.*	25,74 *ſubd.*
305,37 *add.*	21,4	43,4223	282,96
987,4037			

„ Decimales numeri per integros vel per alios decimales multiplicantur ac ſi forent integri, ut vides.

72,4	50,18	3,9025
29	2,75	0,0182
6516	25090	78050
1448	35126	117075
2099,6	10036	39025
	137,9950	0,05451300

Sed

Sed nota quod *in prodeunte numero, tot ſemper figuræ ad dextram pro decimalibus abſcindi debent, quot ſunt figuræ decimales in utroque numero multiplicante.* Et ſi forte non ſint tot figuræ in prodeunte numero, deficientes loci circulis adimplendi ſunt ut vides in exemplo tertio.

„ Quod ſi reſiduum a diviſione nume-
„ ri integri per integrum minus placet
„ poſſis diviſionem in fractionibus deci-
„ malibus ultra ad libitum proſequi, ſem-
„ per adnectendo circulum numero re-
„ ſiduo " ſeu quod eodem redit, adnectendo dividuo integro circulos quosvis ante operationem. Sic $\frac{4798}{23} = 208, 6086$ &c ſi dividuo quatuor circulos adnectas.

Ad eundem modum factio decimalis 3, 5218 per fractionem decimalem 46, 1 dividitur, & prodit 0, 07639 &c. ubi nota quod *in quoto tot figuræ pro decimalibus abſcindendæ ſunt, quot ſunt in ultimo dividuo plures decimales quam in diviſore*, ut in

```
46, 1) 3, 5218 (0, 07639
        3227
        ----
         2948
         2766
         ----
          1820
          1383
          ----
          4370
```

In hoc exemplo quinque, quia ſex ſunt in ultimo dividuo 0, 004370 & una in diviſore 46, 1.

$$\text{Porro } \frac{2099,6}{72,4} = 29 \text{ \& } \frac{137,995}{50,18} = 2,75 \text{ \& } \frac{0,051513}{0,0132} = 3,9025.$$

In extractione radicum ſi veram radicem magis appropinquare placet proſequenda eſt operatio in decimalibus numeris, adnectendo ad reſiduum duos circulos in ſingulis operationibus. Ita radix numeri 22178791 prodit 4709, 43637 &c.

Ubi vero radix ad medietatem aut ultra extracta eſt, cœteræ figuræ per ſolam diviſionem obtineri poſſunt. Ut in hoc exemplo ſi radicem usque ad novem figuras extrahere animus eſſet, poſtquam quinque priores 4709, 4 extractæ ſunt, quatuor poſteriores 3637 elici poſſunt dividendo reſiduum 34264 adnexis quatuor circulis, per duplum 4709, 4 ſeu 94288. (25. Th. I.). Ita radix ex 32976 prodit 181, 59.

Eadem methodo radices etiam è decimalibus numeris extrahuntur. Sic ex 329, 76 radix eſt 18, 159. Et ex 3, 2976 radix eſt 0, 18150. Et ſic præterea. Sed

ex

ex 3297, 6 radix eſt 57, 4247. Et ex 33, 976 radix eſt 5, 74247. Atque ita ex 9, 9856 radix eſt 3, 16. Sed ex 0, 99856 radix eſt 0, 999279 &c. Ut ex ſubjectis diagrammatis conſtare poteſt.

32'97; 6(57, 4247 &c.	0; 99' 85' 6 (0, 999279. &c.
25	81
797	1885
749	1701
4860	18460
4576	17901
1148) 284000 (247	1998) 559000 (279.

55. Binomium decimale $q + 0,1$ elevatur facile ad quemcumque gradum, puta hic ad cubum (per 34. Th. VI.). Nam pone $q + 1$.

fiet $q^3 + 3q^2 + 3q + 1$

x 1. 0, 1. 0, 01. 0, 001

cubus . . . $q^3 + 0,3q^2 + 0,03q + 0,001$.

Q. E. I.

RESOLUTIO FINALIS ÆQUATIONUM OPE LIMITUM.

56. In ea 1°. quæro Limites inter quos radices latent (49). Eſto ex. gr. $x^2 - 16x + 55 = 0$, in qua poſito $x = 0$, reſultat +. Ergo radix minor latet inter limites 0 & ſemiſummam 8. ſubſtituo pro x numerum intermedium 4 & reſultat +. Ergo $4 < x$ lateteque radix minor inter 4 & 8.

4 & 8. Pono $x = 6$ & reſultat $-$. Ergo $x < 6$ latetque radix inter 4 & 6. Ergo ſubſtituo 5 pro x & reſultat 0; hinc $x = 5$. (*def.* IV) Eodem modo detegetur radix major 11 latens inter 8 & $16 + 1 = 17$.

Nunc proponatur $x^3 - 15x^2 + 63x - 50 = 0$. Eſt (per 49) æquatio limitum $3x^2 - 30x + 63 = 0$. ſeu $x^2 - 10x + 21 = 0$, cujus radices 3 & 7. Ergo 1ª radix propoſitæ latet inter 0 & 3; 2ᵈᵃ inter 3 & 7. &c. In ea ſubſtituto 0 pro x reſultat $-$, tum 1 pro x, reſultat $-$, ſed ſubſtituto 2 pro x reſultat $+$. Ergo latet x inter limites 1 & 2 & illa radix erit numerus ſurdus. Cujus valor ut quam proximus vero fiat.

Pono $x = 1 + f$ & ſubſtituo $1 + f$ pro x in propoſita.

$$\begin{aligned} x^3 &= 1 + 3f + 3f^2 + f^3 \\ -15x^2 &= -15 - 30f - 15f^2 \\ +63x &= 63 + 63f \\ -50 &= -50 \end{aligned}$$

Hinc $x^3 - 15x^2 + 63x - 50 = 0 = -1 + 36f - 12f^2 + f^3$.

Cum vero ſit $f < 1$ & quo magis elevetur fractio eo minoris ſit valoris, patet negligi poſſe terminos f^3 & $-12f^2$, ut fiat reſiduum $36f - 1 = 0$ ſeu $f = 1 : 36 = 0.027$. Itaque $x = 1,027$ proxime.

Sed

Sed ut propius accedas, pone $1 = 36f - 12f^2 + f^3$ & erit $f = 1 : (36 - 12f + f^2)$ in cujus denominatore ſcripto $1 : 36$ pro f, erit $f = 1296 : 46225 = 0,02803$.

Longe exactius adhuc determinabis f, ponendo g eſſe differentiam inter illius valorem verum & proximum, ut jam ſit $f = 0,02803 + g$. Nam hoc ſubſtituto in $f^3 - 12f^2 + 36f - 1 =$ ſit

$$f^3 = 0,000022026 + 0,002357g + 0,08409g^2 + g^3$$
$$-12f^2 = -0,00942816 - 0,67272g - 12g^2$$
$$+36f = 1.00908 + 36g.$$
$$-1 = -1$$
$$0 = 0,0003261374 + 35,329637g - 11,9195g^2 + g^3$$

Ex qua neglectis binis ultimis terminis oritur $g = 0,0003261374 : 35,329637 = 0,00000923127$. ita ut ſit $f = 0,02803923127$ & $x = 1 + f = 1,02803923127$ quam proxime. Hunc valorem, $+ h$ ſubſtitue in ultima transformata, ſi vis propius adhuc ad verum accedere &c.

Radix ſecunda latet inter proximos limites 6 & 7. Hinc poſito $x = 6 + f$ eoque valore ſubſtituto pro x reſultabit $f^3 + 3f^2 - 9f + 4 = 0$. Ergo $f = 4 : 9$ proxime.

Sed

Sed $f = 4 : (g-3f-f^2) = 324 : 605$, poſito in diviſore $4 : 9$ pro f.

Hinc $x = 6 + 324 : 605$ proxime &c.

57. Si proponatur $x^3 - 2x - 5 = 0$ cujus radicum una (56) latet inter limites 2 & 3, pono $x = 2 + f$, fitque . . .

$$\begin{aligned} x^3 &= +8 + 12f + 6f^2 + f^3 \\ -2x &= -4 - 2f \\ -5 &= -5 \end{aligned}$$

Ergo $0 = -1 + 10f + 6f^2 + f^3$. & neglectis $6f^2 + f^3$ (ob minutiam) fit $10f = 1$ & $f = 1 : 10 = 0,1$. Hinc pono $f = 0,1 + g$, fitque (   )

$$\begin{aligned} f^3 &= +0,001 + 0,03g + 0,3g^2 + g^3 \\ +6f^2 &= +0,06 + 1,2g + 6g^2 \\ +10f &= +1,0 + 10g \\ -1 &= -1 \end{aligned}$$

Ergo $0 = 0,061 + 11,23g + 6,3g^2 + g^3$. Et $g = -0,061 : 11,23 = -0,0054$. Si placet pono $g = -0,0054 + h$, quo valore ſubſtituto pro g in $g^3 + 6,3g^2 +$ &c. tandem exit $2,09455147$ valor proximus radicis quæſitæ.

Si æquationis $x^3 - 15x^2 + 63x - 50 = 0$, tres ultimos terminos (15) reſolviſſem duplo plures figuras quotienti lucratus fuiſſem; nam pro $f = 1 : 36 = 0,027$ prodiiſſet $f = 0,028031$.

58. COR. „ Eadem methodo poteſt „ extrahi quæcumque radix n ex quo„ cumque monomio numerali A", nempe, ponendo k eſſe valorem radicis proximum in numeris integris & $k+f$ eſſe valorem verum. Nam eſto $k^n + nk^{n-1}f + n \times \frac{n-1}{2} k^{n-2} f^2$ &c $=$ A. Et uſurpatis dumtaxat duobus primis terminis fit $f = (A - k^n) : nk^{n-1}$ ſeu $f = m : nk^{n-1}$, ſcripto nempe m pro cognito $A - k^n$. Sed uſurpatis tribus primis, fit $f = m : (nk^{n-1} + n \times \frac{n-1}{2} k^{n-2} \times m : nk^{n-1})$ $= m : (nk^{n-1} + \frac{n-1}{2k} m$. Ergo $f = km : (nk^n + \frac{n-1}{2} m)$.

Ut ſi quæratur radix cubica numeri 2, hæc erit major quam 1, minor quam 2, hinc pono $k = 1$, & $f = km : (nk^n + \frac{n-1}{2} m$ $= \frac{1}{4} = 0,25$. Dein pono $k = 1,25$ & denuo detego $f = 0,09921$. Adeoque $\sqrt[3]{2} = 1,259921$ proxime.

59. COR. Omnis æquatio cubica 2do termino carens, in qua q negativum, reduci poteſt per $\div 1 \pm \sqrt{q}, q, \pm q\sqrt{q}$ (31. Th. VI.) ad ſequentem formam, $x^3 - x - r = 0$, cujus radices ſunt $-\sqrt{\frac{1}{3}}$, $-\sqrt{\frac{1}{3}}$ & $2\sqrt{\frac{1}{3}}$, ubi $r = \sqrt{\frac{4}{27}}$. Hinc ubi $r > \sqrt{\frac{4}{27}}$ duæ ra-

radicum ſunt impoſſibiles (50.). At ubi $r < \sqrt{\frac{4}{27}}$, tres radices ſunt poſſibiles & inæquales lateque radix affirmativa inter limites 1 & 2. (confer CLAIRAUT El. d'Alg. *pag.* 296. &c) Ea methodus generaliter applicari poteſt omnibus æquationibus cubicis & biquadraticis (52.) eſtque omnium in praxi facillima. Quippe radix transmutatæ ducta in $\sqrt{q}$ erit radix propoſitæ. (34. Th. VI.)

60. Nunc ex numeralium ad litteralium æquationum ſolutionem tranſeamus. Sed prius videndum eſt quomodo reductiones quantitatum litteralium in ſeries infinitas fiant, tum per diviſionem, tum per extractionem radicum.

Eſto fractio $aa:(b+x)$ ſeu a^2 dividenda per $b+x$. Pro aa poſſis ſcribere 1 & $1+1$ pro $b+x$.

$$
\begin{array}{l}
1+1)\,1+0\,(1-1+1-1+1\ \&c.\\
\quad\ \ 1+1\\
\quad\ \ \overline{0-1}\\
\quad\quad -1-1\\
\quad\quad \overline{\ 0+1}
\end{array}
$$

Nunc duc in terminos quoti (*per* 34. Th. VI.) $\div$ onem $\frac{aa}{b}$, $\frac{aax}{bb}$, $\frac{aaxx}{b^3}$ eritque

 aa

$$\frac{aa}{b+x} = \frac{aa}{b} - \frac{aax}{bb} + \frac{aaxx}{b^3} - \frac{aax^3}{b^4}, \text{ \&c.}$$

Q. E. I. Ubi nota 2dum diviſoris terminum x ſeriei numeratores, primum vero b denominatores afficere.

Pari modo fractio $\frac{1}{1+xx}$ mutabitur in $1 - x^2 + x^4 - x^6$ &c. vel in $x^{-2} - x^{-4} + x^{-6} -$ &c. Et fractio $(2x^{\frac{1}{2}} - x^{\frac{3}{2}}) : (1 + x^{\frac{1}{2}} - 3x)$ fiet $2x^{\frac{1}{2}} - 2x + 7x^{\frac{3}{2}} - 13x^2 + 34x^{\frac{5}{2}} - 73x^3$, &c.

Hinc opportune nota, quod ego pro $\frac{1}{x}$, $\frac{1}{xx}$, $\frac{1}{x^3}$, ſcribo x^{-1}, x^{-2}, x^{-3} &c. & $x^{\frac{1}{2}}$, x^3, $x^{\frac{5}{2}}$, pro $\sqrt{x}$, $\sqrt{x^3}$, $\sqrt{x^5}$ &c. ac $x^{-\frac{1}{2}}$, $x^{-\frac{2}{3}}$ $x^{-\frac{1}{4}}$ &c, pro $1 : \sqrt{x}$, $1 : \sqrt[3]{xx}$, $1 : \sqrt[4]{x}$ &c, idque nixus Analogia, ut vides in $\div$one x^2, $x^{\frac{3}{2}}$, x^1, x^0, $x^{-\frac{1}{2}}$, x^{-1}, $x^{-\frac{3}{2}}$.

Eodem pacto pro $a^2 : x, - a^2 b : x^2, + a^2 b^2 : x^3$ &c. ſcribi poteſt $a^2 x^{-1}$, $- a^2 b x^{-2} + a^2 b^2 x^{-3}$, &c; ut & $(a^2 - x^2)^{\frac{1}{2}}$ pro $\sqrt{a^2 - x^2}$ & $(a^2 - x^2)^2$ pro quadrato ipſius $a^2 - x^2$, & $\left(\frac{ab^2 - y^3}{by + y^2}\right)^{\frac{1}{3}}_{\frac{1}{2}}$ pro $\sqrt[3]{(ab^2 - x^2)} : \sqrt{by + y^2}$ &c.

Radix quadrata poteſt extrahi ex $aa + xx$, poſito primum terminum quoti eſſe

eſſe a & notando $1+1$ pro $aa+xx$. Nam

$$1+1\left(1+\tfrac{1}{2}-\tfrac{1}{8}+\frac{1}{16}-\frac{5}{128}+\frac{7}{256}-\frac{21}{1024}\,\&c.\right.$$

$$\frac{1}{0+1}$$

$$\frac{1+\tfrac{1}{4}}{0-\tfrac{1}{4}}$$

$$\frac{-\frac{1}{4}-\frac{1}{8}+\frac{1}{64}}{0+\frac{1}{8}-\frac{1}{64}}$$

$$\frac{+\frac{1}{8}+\frac{1}{16}-\frac{1}{64}+\frac{1}{256}}{0-\frac{5}{64}+\frac{1}{64}-\frac{1}{256}\ \&c.}$$

$$\frac{-\frac{5}{64}-\frac{5}{128}+\frac{5}{512}}{0+\frac{7}{128}-\frac{7}{512},\ \&c.}$$

$$\frac{+\frac{7}{128}+\frac{7}{256},\ \&c.}{0-\frac{21}{512},\ \&c.}$$

Jam duc a in primum quoti terminum & reliquos (ut 34. Th. VI.) in ÷onem

$\frac{xx}{a}, \frac{x^4}{a^3}, \frac{x^6}{a^3}$ &c, eritque $a + \frac{xx}{2a} - \frac{x^4}{8a^3}$, &c $= \sqrt{aa + xx}$ (Q. E. I.) & mutatis ſignis alternis; $= \sqrt{aa - xx}$.

Pariter radix ipſius $x - x^2$ erit $x^{\frac{1}{2}} - \frac{1}{2} x^{\frac{3}{2}} - \frac{1}{8} x^{\frac{5}{3}} - \frac{1}{16} x^{\frac{7}{2}}$, &c. Et ipſius $aa + bx - xx$ erit $1 + \frac{1}{2} - \frac{1}{8}$, ut ſupra, ducto a in primo quoti termino & reliquis in ÷nem $\frac{bx - xx}{a}$, $(\frac{bx - xx}{a_3})^2$ &c.

Hinc patet $\sqrt{1 + ax^2} : \sqrt{1 + bx^2}$ eſſe $(1 + \frac{1}{2} ax^2 - \frac{1}{8} a^2 x^4$, &c.$) : (1 - \frac{1}{2} bx^2 - \frac{1}{8} b^2 x^4$, &c), quamquam ſæpius abbrevietur opus extrahendo radicem ſolius numeratoris, prius in denominatorem ducti, & eam per ipſum dividendo.

Ita puto ſatis apparet modus extrahendi alias quaſvis radices & quomodo ad infinitas terminorum ſimplicium series revocari valeant quantitates quantumvis Radicalibus aut denominatoribus implicatæ, ut $x^3 + (\sqrt{x - \sqrt{1 - xx}}) : \sqrt[3]{ax^2 + x^3})$ $-(\sqrt[5]{x^3 + 2x^5 - xx - x^{\frac{3}{2}}}) : \sqrt[3]{x + x^2 - \sqrt{2x - x^{\frac{2}{3}}}}$.

61. His præmiſſis nota, quod omnis æquatio duas tantum litteras involvens in numeralem poſſit reduci per (34. Th. VI.)

Si

Si æquatio reſolvenda plures duabus litteras involvit, ut $x^3 + a^2x - 2a^3 + ayx - y^3 = 0$ tunc exprimi poterit valor ipſius x per ſeriem cujus terminorum factores & diviſores erunt dimenſiones ipſorum a & y. Ex. gr. pone $x = y + \frac{y^2}{a} + \frac{y^3}{aa} + \frac{y^4}{a^3} + \frac{y^5}{a^4}$, &c & $y < a$; eo citius decreſcent termini ſeriei quo magis y differt ab a; quippe ſi $a : y = y : \frac{y^2}{a}$ & y evaneſcat reſpectu ipſius a, evaneſcet $\frac{y^2}{a}$ reſpectu ipſius y & $y^3 : aa$ reſpectu præcedentis $y^2 : a$ &c. Eadem ratione poſito $y > a$ & $x = a + \frac{aa}{y} + \frac{a^3}{yy}$ &c eſſeque y longe majus quam a; ſeries jugiter decreſcet, ut pro ipſo valore proximo radicis aliquot ex primis terminis aſſumere liceat.

Vis æquationis $x^3 + a^2x - 2a^3 + ayx - y^3 = 0$, pro radice x ſeriem, quæ eo citius decreſcat quo y minus eſt reſpective ad a; pone y evanuiſſe & de reſiduo $x^3 + a^2x - 2a^3 = 0$, extrude $x = a$, quod omnino verum foret ſi $y = 0$; verum y non eſt abſolute nihil, ſed reſpective ad a. Sic preſſio minima per contrariam preſſionem

nem ingentem evaneſcit. Hinc cum ſit a limes proximus valoris x, pono (ut 56.) $x = a + p$; quo valore ſubſtituto in $x^3 + a^2 x - 2a^3 + ayx - y^3 = 0$ reſultat $0 = 4a^2 p + a^2 y + 3ap^2 + apy + p^3 - y^3$. E quibus terminis, quia p & y ex hypotheſi ſunt perexigui valoris, uſurpo tantum maximos reſpective $4a^2 p + a^2 y = 0$, ſitque $p = -\frac{1}{4} y$. Ergo $x = a + p = a - \frac{1}{4} y$ proxime.

Ut propius accedam pono $p = -\frac{1}{4} y + q$, quo valore (ut 56.) ſubſtituto in $p^3 + 3ap^2 + 4a^2 p + apy + a^2 y - y^3 = 0$ reſultat $\frac{65}{64} y^3 - \frac{1}{16} ay^2 + 4a^2 q + \frac{3}{16} y^2 q - \frac{1}{2} ayq - \frac{3}{4} yq^2 + 3aq^2 + q^3 = 0$, è cujus terminis, quia $y < a$ & $p < a$ & $q < p$, neglectis reliquis pono $- \frac{1}{16} ay^2 + 4a^2 q = 0$ ſitque $q = \frac{1}{64} \times \frac{yy}{a}$. Hinc $x = a - \frac{1}{4} y + \frac{yy}{64a}$ Et ita proſequendo fit $x = a - \frac{y}{4} + \frac{y^2}{64a} - \frac{131 y^3}{512 a^2} + \frac{509 y^4}{16384 a^3}$ &c; &c.

Si in æquatione $x^3 - a^2 x + ayx - y^3 = 0$, loco y, ſupponas a evanuiſſe, fiet $x^3 - y^3 = 0$, ſeu $x = y$ proxime, & accuratius $x = y + p$. Quo valore ſubſtituto pro x in

in propoſita reſultabit $3y^2p + ay^2 + ayp + 3yp^2 + p^3 - a^2y - a^2p = 0$, in qua neglectis reliquis terminis aſſume maximos $3y^2p + ay^2 = 0$, fietque $p = -\frac{1}{3}a$ proxime. Ergo $x = y - \frac{1}{3}a$ & opere continuato $x = y - \frac{1}{3}a + \frac{a^2}{3y} + \frac{a^3}{81y^2} - \frac{8a^4}{243y^3}$ &c.

Totum artificium eſt in Electione terminorum propoſitæ pro condenda æquatione fictitia ex qua primus ſeriei terminus extruditur. „ Rite facta confetur „ Electio, ſi ſubſtituto illo primo pro ra-„ dice in terminis reliquis non electis „ eleventur ſinguli ad altiores vel depri-„ mantur ad inferiores dimenſiones „ quam termini electi " Hinc in ultima æquatione $x^3 - a^2x + ayx - y^3 = 0$, perverſe elegiſſem $ayx - y^3 = 0$, nam foret $x = y^2 : a$, quo valore ſubſtituto in terminis non electis fit $x^3 = y^6 : a^3$ & $-a^2x = -ay^2$ ſeu $ax = y^2$.

62. Huic terminorum electioni lucem affert Auctoris Parallelogramum BAC (Fig. 8.) in cujus caſis vides dimenſio- TAB. VI. nes ipſius x ab x^0 ad x^4, multiplicatas FIG. 8. & 9. per dimenſiones ipſius y ab y^0 ad y^6. Ita-

quæ propoſita æquatione', „ Aſterico „ nota in fig. 9. caſas reſpondentes omnibus ejusdem terminis & regulam DE „ applicâ duabus aut forte pluribus ex „ caſis ita notatis, quarum una ſit ima „ in columna AB ſiniſtrorſum, altera „ vero contingat regulam dextrorſum „ & reliquæ omnes regulam non tan„ gentes ſint ſupra eam. Tum ſume eos „ æquationis terminos qui caſis à regula „ tactis includuntur & ex iis extrude „ quantitatem in Quotiente ponendam" Sic ſi radix y debeat extrahi ex æquatione $y^6 - 5xy^5 + \frac{x^3}{a} y^4 - 7a^2x^2y^2 + 6a^3x^3 + b^2x^4 = 0$, noto in Fig. 9. caſas pertinentes ad terminos hujus æquationis Aſterico * ut hic factum vides. Dein Regulam DE applico ad infimam caſarum notatarum in ſiniſtra columna & Regulam in gyrum dextrorſum ago ex inferioribus verſus ſuperiora, donec incipiat ſimiliter aliam vel forte plures ex caſis notâ diſtinctis contingere, & video loca ſic tacta eſſe ea quibus includuntur termini x^3, x^2y^2, & y^6. Hinc effingo ex his æquationem $y^6 - 7a^2x^2y^2 + 6a^3x^3 = 0$, quam præterea, ſi placet, redigo per $\div 1$, $\sqrt{ax}$, ax, $ax\sqrt{ax}$ &c

(34.

(84 Th. VI.) ad $y^6 - 7y^2 + 6 = 0$, in qua invenio valorem ipſius y quadruplicem (36.) nempe ± 1 & $\pm \sqrt{2}$, (ſeu poſt reſtitutionem) $\pm \sqrt{ax}$ & $\pm \sqrt{2ax}$ è quibus quemlibet aſſumere poſſum pro Quotientis termino primo; tamen ſeligendus eſt ille qui convenit huic aut illi radici quam extrahere cupio.

Sic propoſita æquatione $y^5 - by^2 + 9bx^2 - x^3 = 0$, ſegrego terminos $-by^2 + 9bx^2$ & hinc habeo $y = 3x$ pro Quoti termino primo.

Et ſi daretur $y^3 + axy + a^2y - x^3 - 2a^3 = 0$, (quæ eſt prima quam (61.) reſolvimus, ordinata juxta dimenſiones litteræ y..) acciperem $y^3 + a^2y - 2a^3$ cujus radix eſt a quam quotienti adſcribo, foretque $y = a - \frac{x}{4} + \frac{xx}{64} + \frac{131x^3}{512aa} + \frac{509x^4}{16384a^3}$, &c.

Pariter ſi haberem $x^2y^5 - 3c^4xy^2 - c^5x^2 + c^7 = 0$, excerperem $x^2y^5 + c^7$, unde obtinerem $y = -\sqrt[5]{c^7 : x^2}$ quæ quantitas eſſet primus quoti terminus. Et ita porro.

Verum ſi proponatur æquatio $8z^6y^3 + az^6y^2 - 27a^9 = 0$ cujus radix incipit à ter-

à termino $3a^3 : 2z^2$, eam deprimo ipſâ z^2 antequam reſolutionem aggrediar, ne neceſſe ſit eam in ipſa reſolutione deprimere, ita ut evadat $8z^4y^3 + az^4y^2 - 27a^9 z^{-2} = 0$.

63. Tum opere progrediente reliqui termini quibus quotiens augendus eſt, eodem pacto excuduntur ex variis æquationibus 2dariis, ſed plerumque minori conatu: nam totum negotium perficitur „ dividendo ſummam ex iis terminis qui „ ducuntur in p^o vel q^o vel r^o..& indefi„ nite parvis ſpeciebus (x, x^2, x^3, &c:) „ ſunt affecti, per quantitatem qua affi„ citur quantitas p vel q vel r unius di„ menſionibus & libera ab indefinita „ ſpecie, & quotienti adſcribendo quid„ quid inde conficitur." Sic termini $\frac{x}{4}$, $\frac{xx}{64a}$, $\frac{131x^3}{512aa}$, &c. gignuntur dividendo a^2x, $\frac{ax}{16}$, $\frac{131x^2}{128}x^3$ per $4a^2$ in exemplo ſequenti, cujus reſolutionis praxim jam exhibeo. Eſto Diagramma..

$y^3 + a^2y - 2a^3 + axy - x^3 = 0$	$y = a - \frac{x}{4} + \frac{xx}{64a} + \frac{131x^3}{512aa} + \frac{509x^4}{16384a^3}$. &c.
$+a+p=y.$ $+y^3$	$a^3 + 3a^2p + 3ap^2 + p^3$
$+axy$	$+a^2x + axp$
$+aay$	$+a^3 + a^2p$
$-x^3$	$-x^3$
$-2a^3$	$-2a^3$ A
$-\frac{1}{4}x + q = p.$ $+q^3$	$-\frac{1}{64}x^3 + \frac{3}{16}x^2q - \frac{3}{4}xq^2 + q^3$
$+3ap^2$	$+\frac{3}{16}ax^2 - \frac{3}{4}axq + 3aq^2$
$+apx$	$-\frac{1}{4}ax^2 + axq$
$+4a^2p$	$-a^2x + 4a^2q$
$+a^2x$	$+a^2x$
$-x^3$	$-x^3$ B
$+\frac{xx}{64a} + r = q.$ $+q^3$	*
$-\frac{3}{4}xq^2$	*
$+3aq^2$	$+\frac{3x^4}{4096a}$ * $+\frac{3}{32}x^2r + 3ar^2$
$+\frac{3}{16}x^2q$	$+\frac{3x^4}{1024a}$ * $+\frac{3}{16}x^2r$
$-\frac{1}{2}axq$	$-\frac{1}{128}x^3 - \frac{1}{2}axr$
$+4a^2q$	$-\frac{1}{16}ax^2 + 4a^2r$
$-\frac{65}{64}x^3$	$-\frac{65}{64}x^3$
$-\frac{1}{16}ax^2$	$-\frac{1}{16}ax^2$ C

$4a^2 - \frac{1}{2}ax + \frac{9}{32}x^2)\ \frac{131}{128}x^3 - \frac{15x^4}{4096a}\ (\frac{131x^3}{512aa} + \frac{509x^4}{16384a^3}$

In

In caſa A mox video terminos $-x^3$, $-3ap^2$, & p^3 evaneſcere. Nam ex æquatione fictitia $y^3 + a^2y - 2a^3 = 0$ prodit $y = a$; hinc $x \lessdot y$ vel $x \lessdot a$, alias $axy - x^3$ non potuiſſent evaneſcere. Dein $p \lessdot a$. Ergo reſtat $a^2x + 4a^2p + apx$. Sed ex mox notatis evaneſcit $apx \lessdot a^2x$ & $\lessdot 4a^2p$, adeoque reſtat $a^2x + 4a^2p = 0$, ſeu $p = -\frac{1}{4}x$.

In caſa B obſervo $q \lessdot x$ & $x \lessdot a$. Hinc evaneſcunt omnes termini quorum factores q^3, q^2, qx^2, & qx manente ſolo $4a^2q$, ut & $-x^3$, manente $-\frac{ax^2}{16} > \frac{3qx^2}{16}$. Ergo $4a^2q - \frac{ax^2}{16} = 0$ ſeu $q = \frac{xx}{64a}$.

In caſa C exprimitur r per ſeriem infinitam ope diviſionis. Hinc neglecto $+ 3ar^2$, fit (addendo) $\frac{15x^4}{4096a} - \frac{131x^3}{64} + \frac{9rx^2}{32} - \frac{1}{2}axr + 4a^2r = 0$, ſeu $r \times (\frac{9x^2}{32} - \frac{1ax}{2} + 4a^2) = 131x^3 : 64, - 15x^4 : 4096a$. Ergo $r = 131x^3 : 512a^2, + 509x^4 : 16384a^3$ ut vides ſub caſa C.

Si quotiens ad certam periodum tantum perducendus foret, ita ut ex. gr. x in ultimo termino non aſcenderet altius quam ad dimenſionem x^4, omitterem terminos omnes poſt x^4 & unum ponerem

rem poſt z^3: juxta hanc Regulam. „Pri-
„mo termino qui ſuboritur ex quavis
„quantitate in collaterali margine tot
„ſubjungito terminos verſus dextram
„quot unitatibus index poteſtatis ejus,
„ab indice poteſtatis quam ſtatuiſti al-
„tiſſimam fore in quotiente, ſuperatur."
Quocirca pro deletis habendi ſunt in caſa C omnes termini poſt ſignum *. Et opus ita proſequendo eandem ſeriem obtinebis quam vides ſupra caſam A.

Pariter ſi proponas æquationem $\frac{63}{2816}y^{11} + \frac{35}{1152}y^9 + \frac{5}{112}y^7 + \frac{3}{40}y^5 + \frac{1}{6}y^3 + y - z = 0$; reſolvendam ad nonam uſque quoti dimenſionem, poſito $y = z$ (per 62) rejicere potes ante ipſum operis exordium $63y^{11} : 2816$ & inter operandum omnes terminos qui ſuperant z^9, admittere autem unum poſt z^7, duos poſt z^5; obſervabis enim quotientem pariter aſcenſurum ut $\div z, z^3, z^5, z^7$ &c: Demum autem habebis $y = z - \frac{1}{6}z^3 + \frac{1}{120}z^5 - \frac{1}{5040}z^7$ $\frac{1}{362880}z^9$, &c.

Hinc detegitur artificium quo æquationes affectæ in infinitum, aut conſtantes

tes infinito terminorum numero, tamen reſolvi poſſunt. Ut ſi radix cubica extrahenda ſit ad usque z^4 ex æquatione...

$$0 = \begin{cases} +y^0 \text{ in } -8 + z^2 - 4z^4 + 9z^6 \text{ \&c.} \\ +y \text{ in } zz - 2z^4 + 3z^6 - 4z^8 \text{ \&c.} \\ -y^2 \text{ in } zz - z^4 + z^6 - z^8 \text{ \&c.} \\ +y^3 \text{ in } zz - \frac{1}{2}z^4 + \frac{1}{3}z^6 - \frac{1}{4}z^8 \text{ \&c.} \end{cases}$$

Omitto cunctos in infinitum terminos ſuperiores tribus primis in quacumque ſerie: quare hanc ſolam æquationem reſolvendam aſſumo $\frac{1}{3}z^6y^3 - \frac{1}{2}z^4y^3 + z^2y^3 - z^6y^2 + z^4y^2 - z^2y^2 + 3z^6y - 2z^4y + z^2y - 4z^4 + z^2 - 8 = 0$. nam quoti 1us terminus $(2z^{-\frac{2}{3}})$ ſubſtitutus pro y in ea æquationis depreſſæ per $z^{\frac{8}{3}}$ parte quam neglexímus dat ſemper plus quam quatuor dimenſiones.

Quæ de altioribus diximus poſſunt & quadraticis æquationibus applicari. Ut ſi quæratur radix hujus

$$0 = \begin{cases} y^2 \\ -y \text{ in } a + x + \frac{xx}{a} + \frac{x^3}{aa} + \frac{x^4}{a^3}, \text{ \&c.} \\ + \frac{x^4}{4aa} \end{cases}$$

Usque ad x^6; omitto cunctos in infinitum terminos altiores quam $-y$ in a

$+ x$

$+x+\frac{xx}{a}$ & ſolum aſſumo $y^2-ay-xy-\frac{xx}{a}y$ $+\frac{x^4}{4aa}=0$, quam reſolvo more ſolito (per 15), aut brevius & facilius jam tradita methodo affectarum æquationum qua obtinebis $y=\frac{x^4}{4a^3}-\frac{x^5}{4a^4}$ * in qua ultimus requiſitus terminus evaneſcit aut nihilo fit æqualis.

Radices autem poſtquam extractæ ſunt usque ad ſatis magnum terminorum numerum ulterius quousque libet extrahi poſſunt, ſola ſerierum Analogia bene obſervata. Sic in infinitum proſequi potes hanc $z+\frac{1}{2}z^2+\frac{1}{6}z^3+\frac{1}{24}z^4+\frac{1}{120}z^5$, &c (quæ eſt radix æquationis infinitæ $z=y-\frac{1}{2}y^2+\frac{1}{3}y^3-\frac{1}{4}y^4$ &c.)ſemper dividendo terminum ultimum per hanc numerorum ſeriem 2, 3, 4, 5, 6, 7, &c. Nam $2\times3=6$ & $6\times4=24$. &c.

Sed inveſtigando quoti terminum 1um, vel etiam 2dum vel 3tium, ejus valor poteſt eſſe quantitas ſurda vel inextricabilis ab æquatione fictitia. Quando id forte evenit, radix iſta (modo non ſit impoſſibilis) exponi poteſt per aliquam litteram b, quo facto procede more ſolito ac ſi cognita foret.

Hactenus ſpecies indefinitas parvas eſſe ſuppoſuimus. Si vero quam propius accedere ponerentur datæ quantitati, pro indefinite parva illa differentia ſpeciem aliquam adhibeo, qua ſubſtituta ſolvo æquationem ut antea. Quod ſi ſpecies poneretur indefinite magna, pro ea ſcribe ejus reciprocam, quæ erit infinite parva.

Sin autem artium iſtarum nulla ſuccederet, ut in æquatione $x^4 - 2x^2y^2 + xy^3 + 2y^2 - 2y + 1 = 0$, cujus æquatio fictitia $x^4 + 2y^2 - 2y + 1 = 0$ nullam habet radicem poſſibilem, potes *ex. gr.* ſupponere quod x parum differat à $+2$ vel quod $2 + x = z$, quo valore ſubſtituto pro x invenies $y^4 - z^2y^2 - 3zy^2 - 2y + 1 = 0$, & quotiens incipiet ab unitate. Potes etiam ſupponere quod x eſt quantitas indefinite magna vel quod $\frac{1}{x} = z$, unde deduces

$$y^4 - \frac{yy}{zz} + \frac{yy}{z} + 2y^2 - 2y + 1 = 0$$

& z chorum terminorum ducet in quotiente. Aliis adhuc forté modis voti compos fies.

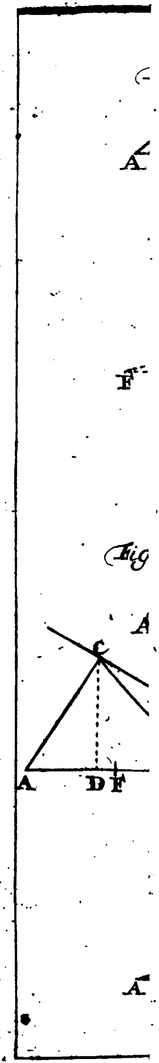
Ut autem pateat veritas hujus concluſionis, nempe quod Quotientes ſic inventi & producti ad libitum adeo prope

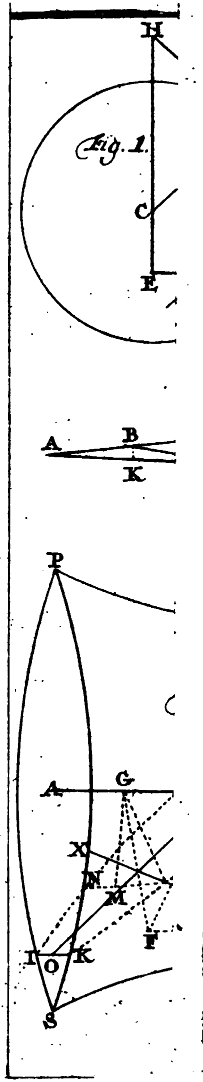
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ad æquationis radicem accedunt quam quævis aſſignabilis & idcirco quod in infinitum protracti omnino ab ea non differunt, nota quantitates quæ ſunt in ſiniſtra columna dextri lateris Diagrammatis eſſe ultimos terminos æquationum quarum radices ſunt *p*, *q*, *r*, *s*, &c & quod illis evaneſcentibus, evaneſcant quoque radices *p*, *q*, *r*, *s*, &c id eſt differentiæ inter Quotientem & Radicem quæſitam; ita ut Quotiens nihil omnino tunc differat à radice vera. Quapropter ſub operis initium, ſi videas quod termini qui ſunt in indicata Columna ſeſe omnes invicem deſtruant, conclude quotientem hucusque protractum perfectam æquationis radicem eſſe; quod ſi res aliter ſe habeat, videbis tamen quod omnes termini in quibus ſpecies indefinite parva eſt paucarum dimenſionum, id eſt, omnes maximi termini auferuntur à Columna iſta, ita ut denique aut nulli reſtent in ea, aut ii tantum qui aſſignata qualibet quantitate minores ſunt & idcirco nihilo non majores, ubi opus eſt in infinitum protractum: Ergo quotiens extractus in infinitum eſt demum ipſiſſima Radix.

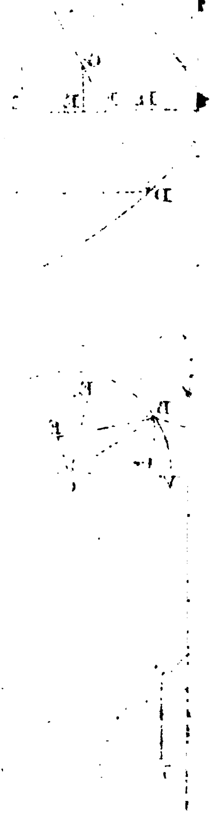
Ab his ſummus *Newtonus* tranſit ad Calculum Fluxionum ſeu differentialem. Nos vero, ſoluta fide §. 1. data, ſubſiſtimus.

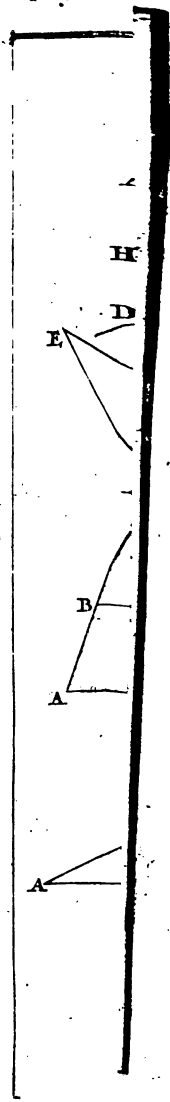
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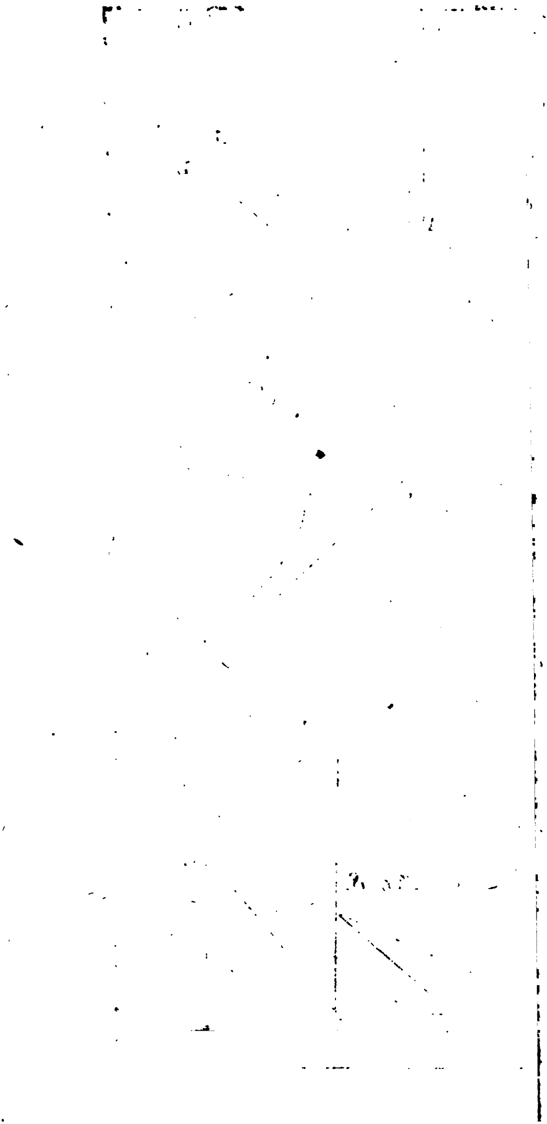

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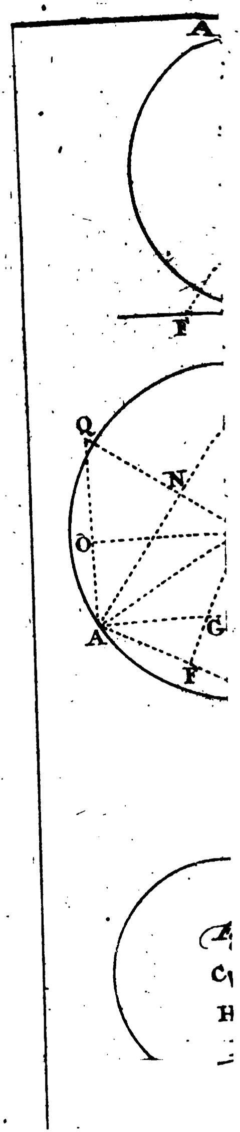


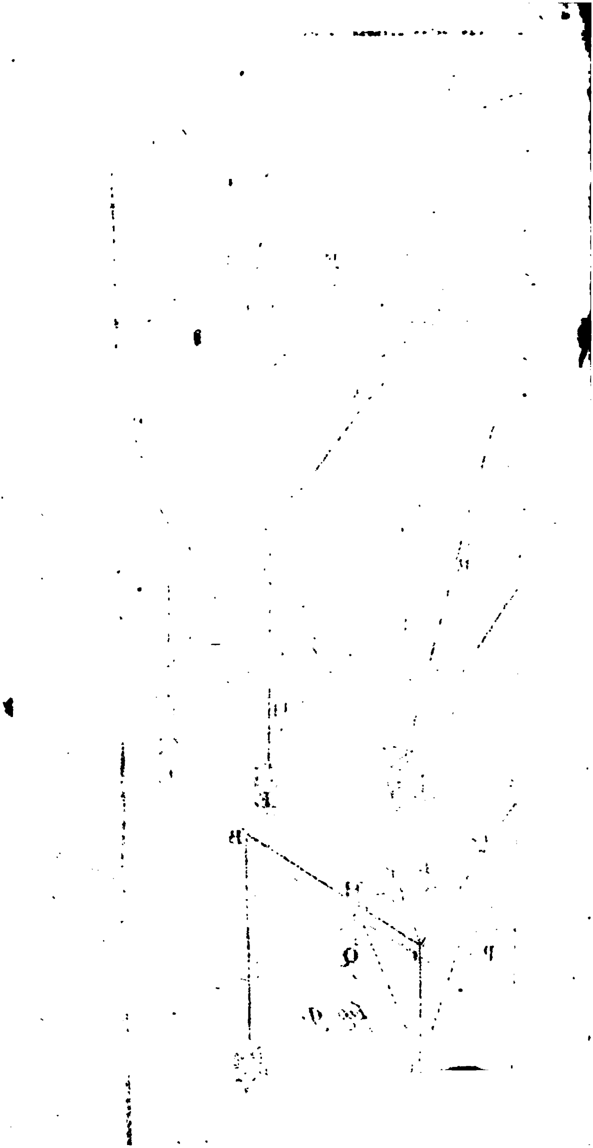
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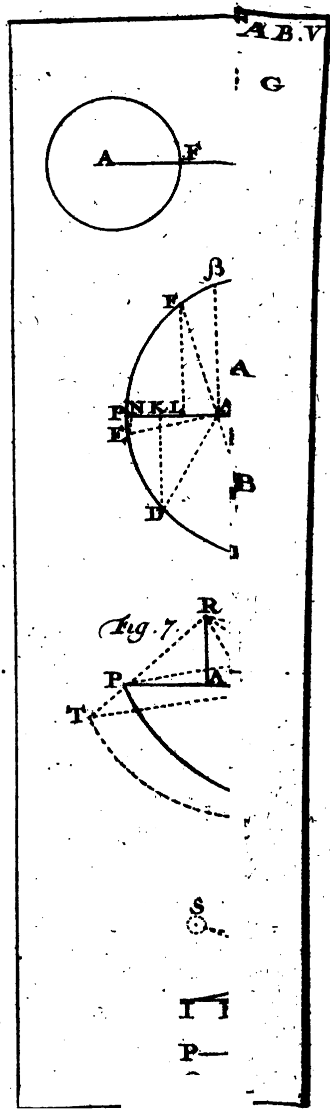


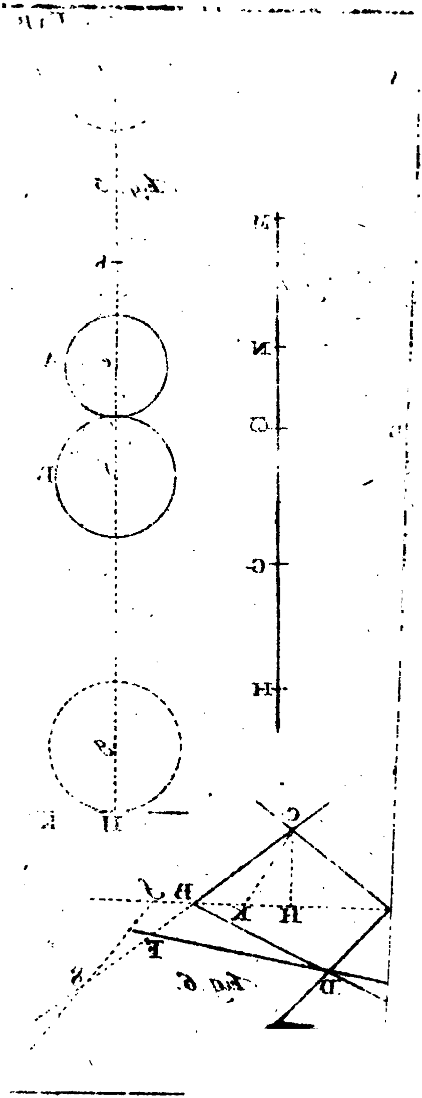

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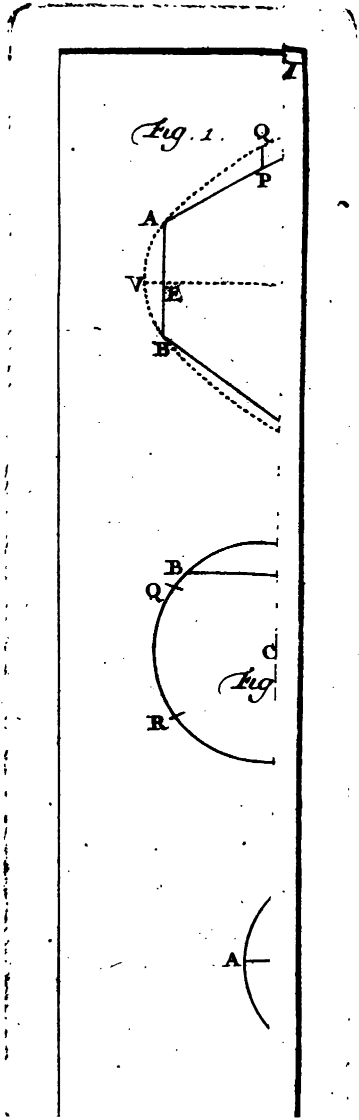



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